NEW SERIES, NO. 148 (VOL. XIX) DECEMBER, 1924

JOURNAL OF THE AMERICAN STATISTICAL ASSOCIATION

Formerly the Quarterly Publication of the American Statistical Association

A METHOD OF HANDLING CURVILINEAR CORRELATION FOR ANY NUMBER OF VARIABLES

By Mordecai Ezekiel, *Bureau of Agricultural Economics, U. S. Department of Agriculture*

Linear correlation analysis is a means of measuring the relations subsisting among two or more variables in so far as the functions are truly, or even approximately, linear. More complicated curvilinear relations have long been recognized, and various methods have been devised for determining them. Little work has yet been done, however, on the problem of simultaneously determining the curvilinear relation of two or more independent variables to a dependent variable.[1]

In many fields of work, the usefulness of any analysis which assumes linear relations between the variables is decidedly limited. Every set of biological relations that includes the curves of growth, every economic study that touches diminishing returns, the demand curve, or the curve of elasticity of supply; every psychological study that includes speed of reaction, involves relations which are decidedly non-linear. Determination of the exact shape of the curve is frequently the most important objective in such studies. Ability to make this determination under "field" conditions, where many factors must be considered simultaneously, would greatly increase the usefulness of correlation analysis.

I

SIMPLE CURVILINEAR CORRELATION

Methods of determining curvilinear regression lines. A broken line connecting group averages serves as a first approximation to the cur-

[1] The method of treating multiple correlation problems discussed in this paper was included in a graduate thesis submitted to the University of Minnesota in June, 1923. The work which resulted in the establishment of this method was done under the direction of Dr. J. D. Black of that institution and Mr. H. R. Tolley of the Bureau of Agricultural Economics. Acknowledgement is due to both them and others at Minnesota and Washington for help in preparing this paper.

vilinear regression. Owing to fluctuations due both to the error in the group-averages, and to the effects of other variables, even with very large groups these averages will seldom define a smooth continuous curve, but will vary enough to make the line more or less uneven. The true relationship would be expected to be that of a smooth continuous function, rather than of the irregular line. Hence some method of smoothing the line of averages must be used. This can be done most simply by free-hand smoothing. In drawing such a line each average may be given equal weight, except where the number of cases is so small that the probable error of the average is very high, or it may be considered as weighted according to the number of cases represented. This line may further be drawn with a view to making the deviations on either axis a minimum, or with a view to making the deviations measured perpendicularly to the line a minimum, depending on the particular result desired.

The possibility of describing regression lines more closely than by linear regressions by fitting parabolae of higher order to the data either by the method of least squares or the method of moments, has long been recognized. Pearson also provides types of curves other than parabolae, and stresses the importance of selecting the proper formula.[1] Types such as logarithmic curves, exponential curves, trignometric curves, and hyperbolic curves are frequently useful for fitting regression lines.[2]

Though fitting a curve mathematically gives more rigid results than free-hand smoothing, there are certain disadvantages. In cases where a certain type of relationship is expected on *a priori* grounds, fitting an equation of the expected type gives a very satisfactory way of testing the agreement between observation and hypothesis. In many cases, however, not enough is known of the relations to express them analytically. In cases of this sort there are serious drawbacks to the use of parabolae or other specific curves. Even though a good fit be obtained, the shape of the curve is conditioned by the equation used. Figure 1 illustrates these objections.

Second and third degree parabolae, and a free-hand line, have been fitted to the data represented by crosses. The cubic parabola gives a closer fit than the free-hand curve, and of course than the second degree parabola, yet obviously the double inflection of the cubic might be entirely contrary to given theoretical considerations.

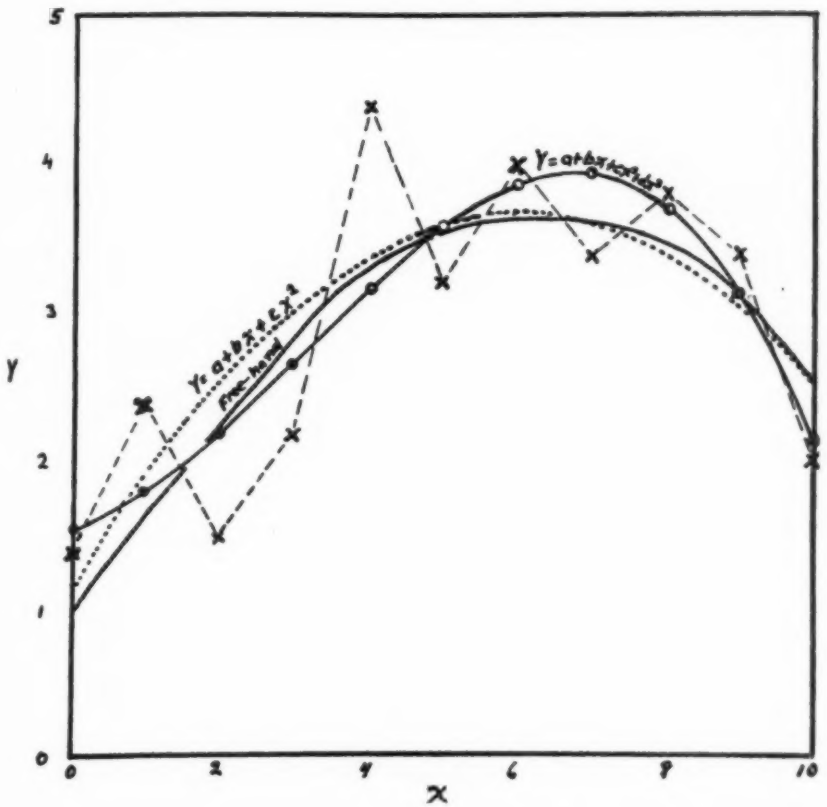
[1] Pearson, Karl: "On the Systematic Fitting of Curves to Observations and Measurements," *Biometrika*, Vol. 1, p. 265; Vol. II, p. 1, 1902.

[2] An interesting example of curve fitting is the application of an exponential curve to the curve of diminishing returns. Spillman, W. J.: "Law of the Diminishing Increment in the Fattening of Steers and Hogs," *Jour. Farm Econ.*, Vol. VI, No. 2, 1924.

The free-hand method affords a way of fitting a curve solely as indicated by the data, with no condition other than that of continuity.

A Criterion for the Closeness of Curvilinear Correlation. The coefficient of correlation, r, measures the closeness with which one variable may be stated as the *linear* function of another. In a similar manner

Fig. 1. Curves Fitted by Various Methods.



the correlation-ratio measures the closeness with which one variable may be stated as a *discontinuous* function of the other, using the means of arrays as the value of the function for each array.

There is need for a definite measure of the closeness with which one variable can be stated as a *continuous curvilinear* function of the other variable. Correcting the value of the correlation-ratio to take account of error in the means and the fineness of grouping is an attempt to do this, but the corrected coefficient lacks definite meaning.

Otis[1] has suggested computing a coefficient of curvilinear correlation by the use of a curvilinear regression graphically determined. His coefficient, however, uses median deviations rather than standard deviations, and hence is not directly comparable with Pearsonian coefficients. The following method is similar to that of Otis; but yields a coefficient directly comparable to the Pearsonian coefficient of correlation.

Construct a curve either graphically or analytically to show the change in y with changes in x. For the observed values of x substitute the estimated values of y given by this curve, which we will designate $y_{(fx)}$. Correlate these $y_{(fx)}$ values with the accompanying values of y, and obtain $r_{yy(fx)}$, which gives the correlation of y with x with respect to the curvilinear regression. The name *index of correlation* is suggested for this measure of curvilinear correlation, and the letter rho, ρ, used to designate it.[2]

The index of correlation is essentially a corrected correlation-ratio, based directly upon the values of y as given by the function of x used, instead of upon the values of y given by the means of arrays. For the mathematical device of correcting the correlation-ratio to try to make the standard deviation of the irregular line of means approximate the standard deviation of points along a continuous curve, the index method substitutes the actual points along the curve, doing away with the necessity of elaborate correction formula.

The procedure for calculating the value of the index of correlation depends upon the kind of curve used. Where a parabola or other function with constants only in the first power is used as the curve, the value of the correlation-index can be determined directly. For since the functional relationship between X, X^2 and X^3 does not prevent considering them as separate variables, the equation

$$Y = a + b_1 X + b_2 X^2 + b_3 X^3$$

reduces to

$$y = b_1 x + b_2 x^2 + b_3 x^3$$

when the variables are stated in terms of deviations from their means.[3] Hence the correlation of Y with values of $Y_{(fx)}$ estimated from the cubic regression curve is identical with the multiple correlation of Y with the three variables X, X^2, and X^3. That is:

$$\rho_{xy} = R_{y \,.\, x\, x^2 x^3}$$

[1] Otis, Arthur S.: "The Reliability of Spelling Scales, including a 'deviation formula' for correlation." *School and Society*, Vol. IV, Nos. 96–99, 1916.

[2] This paper was written before I had seen the article by Dr. Frederick C. Mills, *The Measurement of Correlation and the Problem of Estimation*, published in the preceding issue of this JOURNAL. Since the indexes of correlation developed by each of us independently are mathematically identical, I have altered this paper to follow Dr. Mills' notation.

[3] Tolley, H. R., and Ezekiel, M. J. B.: "A Method of Handling Multiple Correlation Problems," JOURNAL AMERICAN STATISTICAL ASSOCIATION, Vol. XVIII, pp. 995–997, 1923.

Where the curve is not of a type that can be determined by least squares, or where a free-hand curve is used, the values of $Y_{(fx)}$ must be computed separately for each observation, and these values correlated with the values of Y to determine the index of correlation.

The numeric value of the index of correlation will lie between those of the coefficient of correlation and the raw correlation-ratio. For true linear regression, all three will tend to coincide. How closely the value of the correlation-index appreaches that of the correlation-ratio will depend upon how nearly the continuous curve used approaches the broken line of the means of the arrays.

Just as there are two correlation ratios for expressing the closeness of the relation between two variables, so there will be two indexes of correlation, one measuring how closely X can be stated as a continuous function of Y, and the other how closely Y can be stated as a continuous function of X.

Considering the group-averages plotted in Figure 1 as individual observations, the coefficient of correlation of X with Y is $+.47$.[1] If we use five groups, the correlation-ratio is 0.84. Computing the correlation-index for the three curves shown, the following results are obtained:

$$\begin{aligned}
r_{xy} &= +0.47 \text{ (using } Y = a + bX) \\
\rho_{xy} &= \ \ 0.79 \text{ (using } Y = a + bX + cX^2) \\
\rho_{xy} &= \ \ 0.82 \text{ (using free-hand curve)} \\
\rho_{xy} &= \ \ 0.84 \text{ (using } Y = a + bX + cX^2 + dX^3) \\
\eta_{xy} &= \ \ 0.84.
\end{aligned}$$

Though one of the curves fitted analytically gives a higher correlation than the freehand curve, it is obvious that the latter gives a much better measure of the nature and closeness of the relationship between X and Y than does the linear measure.

II

MULTIPLE CURVILINEAR CORRELATION

Only recently has attention been given to the problem of determining the nature of the curvilinear relationships between several variables when all are being considered simultaneously. This problem can be partially solved by eliminating first the apparent effect of one variable, then the effect of the next, and so on. This procedure is open to the objection that in eliminating the apparent or gross effect of one variable, part of the effect of correlated variables is eliminated at the same time. Any valid solution must simultaneously determine the net curvilinear

[1] Probable errors, necessarily large with so few cases, are not shown.

relationship of each independent variable to the dependent variable; some extension of the multiple correlation principle is needed.

McEwen and Michael have offered one means of determining such relationships.[1] Their method defines the function by only a few points, and for more than three or four variables would apparently require a very large volume of calculations. Furthermore, it affords no criterion for judging how closely the original data conform to the functional relations found.

A means of measuring the closeness of correlation among three or more variables by the partial correlation ratio has been suggested by Pearson and developed by Isserlis.[2] Aside from the complexity of the method, the partial correlation-ratio method gives no description of the regression curves, and further, tends to give a higher correlation than could obtain for smooth continuous functions.

Multiple curvilinear correlation by the use of functions of independent variables.—Parabolae of any order, and certain other curves, can be fitted to two-variable problems by the usual methods of least squares. By a simple extension of the same principle functional regressions could, theoretically at least, be fitted to several variables simultaneously. The procedure would be simply to determine the constants of regression equations using such functions of each variable as were desirable.

The regression equations would be as follows:[3]

Linear multiple correlation: $x_1 = b_2x_2 + b_3x_3 + b_4x_4$.

Multiple curvilinear correlation, second power parabolae:

$$x_1 = (b_2x_2 + b'_2x^2_2) + (b_3x_3 + b'_3x^2_3) + (b_4x_4 + b'_4x^2_4).$$

Multiple curvilinear correlation, cubic parabolae:

$$x_1 = (b_2x_2 + b'_2x^2_2 + b''_2x^3_2) + (b_3x_3 + b'_3x^2_3 + b''_3x^3_3) + (b_4x_4 + b'_4x^2_4 + b''_4x^3_4)$$

and so on for higher powers.

Where sufficient observations were available to determine the constants with some degree of accuracy, this method should give the net functional relation of each independent variable to the dependent

[1] Ritter, W. E.: "A Step Forward in the Methodology of Natural Science. (The Functional Relation of One Variable to Each of a Number of Correlated Variables Determined by a Method of Successive Approximation to Group Averages; A contribution to Statistical Methods. By George F. McEwen and Ellis L. Michael.)" *Proc. of the Amer. Acad. of Arts and Sciences*, Vol. 55; No. 2, pp. 91–133, 1919.

[2] Pearson, Karl: "On the Partial Correlation-Ratio;" *Proc Roy. Soc.*, Series A. Vol. XCI, 1915, p. 492; Isserlis, L: "On the Partial Correlation-Ratio, Pt. I, Theoretical," *Biometrika*, Vol. X, 1914, pp. 391–411; "On the Partial-Correlation Ratio, Pt. II, Numerical," *Biometrika*, 1916–1917, Vol. XI, p. 50.

[3] The notation used for partial correlation follows that of G. U. Yule: *An Introduction to the Theory of Statistics*, 6th Ed., pp. 229–252. The possibility of using parabolae in multiple correlation was suggested by Yule in 1897; "On the Theory of Correlation," *Jour. Roy. Stat. Soc.*, Vol. lx, p. 812.

variable x_1, and a measure of the multiple "parabolic" correlation. In harmony with the use for two-variable curvilinear correlation, the term *index of multiple correlation* is suggested for this measure, and the capital rho, P, used to designate it. Then
$Rx_1 \,.\, x_2x^2_2,\ x_3x^2_3,\ x_4x^2_4 = Px_1.x_2x_3$ if second power parabolae are used.

To obtain an *index of net* or *partial correlation*, however, it would be necessary to determine how closely the complete function of each variable is correlated with the dependent variable, after eliminating the functional effects of the other variables. This could be done by computing the index of multiple correlation with all the powers of one variable omitted from the variables, and then determining the index of net correlation from the relation between the index of multiple correlation for three variables and for all the variables,

$$\text{for } 1-R^2_{1\cdot 23\,\ldots\,n} = (1-r^2_{12})(1-r^2_{13\cdot 2})(1-r^2_{14\cdot 23})\ \ldots\ (1-r^2_{1n\cdot 23\,\ldots\,n-1})$$

$$\text{and } 1-R^2_{1\cdot 23\,\ldots\,(n-1)} = (1-r^2_{12})(1-r^2_{13\cdot 2})(1-r^2_{14\cdot 23})\ \ldots\ (1-r^2_{1(n-1)\cdot 23\,\ldots\,(n-2)})$$

$$\text{hence } 1-R^2_{1\cdot 23\,\ldots\,n} = (1-R^2_{1\cdot 23\,\ldots\,n-1})(1-r^2_{1n\cdot 23\,\ldots\,(n-1)})$$

$$\text{by analogy, } 1-P^2_{1\cdot 23\,\ldots\,n} = (1-P^2_{1\cdot 23\,\ldots\,n-1})(1-\rho^2_{1n\cdot 23\,\ldots\,n-1})$$

$$\text{and } \frac{1-P^2_{1\cdot 23\,\ldots\,n}}{1-P^2_{1\cdot 23\,\ldots\,n-1}} = 1-\rho^2_{1n\cdot 23\,\ldots\,n-1}.$$

$$\text{For four variables } \frac{1-P^2_{x_1\,\cdot\,x_2x_3x_4}}{1-P^2_{x_1\,\cdot\,x_2x_3}} = 1-\rho^2_{x_1x_4\,\cdot\,x_2x_3}.$$

That is, where second power parabolae are used,

$$\frac{1-R^2_{x_1\cdot(x_2x_3)(x_2x_3)(x_4x_4)}}{1-R^2_{x_1(x_2x_3)(x_2x_3)}} = 1-\rho^2_{x_1x_4\,\cdot\,x_2x_3}.$$

Multiple regression equations could likewise be obtained using other functions of the independent variables, such as reciprocals, roots, logarithms, and so on; and indexes of multiple and net correlation could be computed for such functional relations. A multiple regression equation could be determined for any function of the independent variables whose constants were all in the first power; and different kinds of functions could be combined if desired. A relatively simple illustration of such a complex functional regression equation would be the following:

$$x_1 = b_2x_2 + b'_2x^2_2 + b_3\left(\frac{1}{x_3}\right) + b_4(\log x_4) + b_5\sqrt{x_5}.$$

The determination of net curvilinear regressions by the use of parabolae or other functions is open to the objection already stated with

regard to determining gross (zero order) regressions analytically: The final curve obtained is conditioned by the specific function used. A further practical objection arises from the fact that determining the net curvilinear regressions in terms of a second degree parabola would double the number of factors in the partial correlation analysis; in terms of a cubic parabola, would triple them; and so on. Determining net cubic regressions for a problem of three independent factors would be equivalent to computing net linear regressions for nine independent factors. Besides the great increase in the work of computing the moments and solving the normal equations, this would mean that the method could not be used unless the number of observations was quite large. For example, if cubic regressions were obtained for five independent variables from 15 observations, a perfect "fit" would be obtained regardless of the observations, as there would be but 15 simultaneous equations from which to determine the value of 15 constants. This would limit the use of the method particularly in the analysis of time-series, where only a limited number of observations are available.

Though it is theoretically possible to determine net regression curves by the use of parabolae or other functions, for the reasons stated it does not seem probable that such a method will prove of much value. Hence other means of accomplishing the result are needed.

DETERMINATION OF NET REGRESSION CURVES BY SUCCESSIVE APPROXIMATIONS

Theoretical basis. In linear multiple correlation the method of least squares is applied to a series of observations X_1, X_2, X_3 . . . X_n, to obtain the constants in a linear regression or "predicting" equation of the form:

$$X' = a + b_2X_2 + b_3X_3 + \ldots + b_nX_n.$$

Having obtained these constants, a residual may be computed for each formula by the equation—

$$\begin{aligned} e &= X_1 - X'_1 \\ &= X_1 - (a + b_2X_2 + b_3X_3 + \ldots + b_nX_n). \end{aligned} \tag{1}$$

Each residual shows the extent to which the observed value of X_1 differs from the value computed by the regression equation.

Now when the true regression of X_1 on X_2 is curvilinear rather than linear, all that the above equation can take into account is a linear relation representing something like the weighted average slope of the curve. At values of X_2 where the curve lies above this straight line, a part of the contribution of X_2 to the value of X_1 will be larger than

the linear estimate of the contribution as given by the regression equation, and hence e will tend to be large positively, and at values of X_2 where the curve lies below the straight line, the contribution of X_2 to the value of X_1 will not be as large as the estimate of the contribution as given by the regression equation, and e will tend to be large negatively.

In a case where the net regressions of X_1 on each factor except X_2 were linear, values of X_1 estimated from the linear regression equation would differ from the observed values for two causes: (1) The effects on X_1 of factors other than those taken into account in the regression equation; and (2) differences between the true curvilinear effect of X_2 on X_1 and the linear effect estimated. If the observations were arrayed with regard to X_2, and the differences between the estimated and observed values of X_1 (the residuals) averaged for each array, these averages would show the curvilinear regression of X_1 on X_2 in terms of deviations from the net regression line. This is true because the part of the differences due to factors not related to X_2 to X_n would tend to cancel out, and only the part due to the regression of X_1 on X_2 differing from a straight line would remain in the averages. Drawing a graph of the net regression of X_1 on X_2, and plotting these average residuals as deviations from that line, with the corresponding values of X_2 as abscissae, would give an indication of the net curvilinear regression of X_1 on X_2, since to the linear regression this would add corrections for lack of linearity. A curve drawn through the means of residuals, plotted as described, would give the net curvilinear regression of X_1 on X_2.

In a case where several of the regressions were curvilinear the causes of differences between values of X_1 estimated from the linear regression equation and the observed values would be more complex. In this case the differences would be due to—(1) the effect upon X_1 of variables not related to those included in the regression equation; and (2) the inability of the several linear regressions to represent the curvilinear relations.

If the observations were arrayed with regard to X_2, and means of residuals taken for each array, the part of the residuals due to (1) would tend to average out, and the means would be the average of the part of the differences due to (2). Besides the failure of the net linear regression of X_1 on X_2 to account for the true curvilinear regression of X_1 on X_2, the means of residuals arrayed with regard to X_2, would also reflect failure of the other net regressions to account for curvilinear regression of X_1 on the other variables. The extent to which these latter components would mask the difference due solely to curvilinearity in

the regression of X_1 on X_2 would depend upon the inter-correlation of X_2 with X_3 to X_n, and the similarity of the curves of regression.

It seems probable that in most cases elements due to variables other than the one upon which the residuals were arrayed would be sufficiently diverse so that the means of residuals arrayed according to X_2 would tend to show the curvilinearity of the relation of X_1 to X_2, even though somewhat masked by the other components. Plotting means of these arrays from the regression line as before, and fitting a curve to them would yield a curve that could be regarded as a first approximation toward the true curvilinear regression. Arraying the residuals upon X_3, and fitting a curve to the means of arrays plotted as deviations from the net regression line of X_1 on X_3 would likewise give a first approximation to the curvilinear regression of X_1 on X_3; and so on in turn for each of the other variables.

Having obtained in this manner curves showing the approximate curvilinear regressions of X_1 on each independent variable, new residuals (e') could be computed by the use of the formula:

$$e' = X_1 - [(K' + f'(X_2) + f'(X_3) + \quad + f'(X_n)] \tag{2}$$

$f'(X_2)$, $f'(X_3)$, etc., representing values read from the first approximation curves, and K' a constant to be determined so as to make $\Sigma(e')^2$ a minimum.

Where the first residuals included components due to the failure of the net regression lines to represent the true curvilinear relations, then second residuals would include components due to failure of the first approximation curves to represent the true curvilinear relations. Since the use of even approximate curves should reduce the average size of the residuals, these second residuals, arrayed in turn on each independent variable, would indicate more definitely what corrections should be made in the curves for each regression. Plotting the means of the residuals for each array as deviations from the approximate regression curve of X_1 on each accompanying variable, and fitting new curves, would give a second approximation to the true regression curves.

These second approximation curves could be used to compute third residuals, e'', by the formula:

$$e'' = X_1 - [K'' + f''(X_2) + f''(X_3) + \quad + f''(X_n)] \tag{3}$$

and the new residuals used to still further correct the curves. This process of successive approximation could be carried through as many stages as necessary to give constant form to the curves.

The determination of net regression curves by a method based on these considerations has been tested in several cases where hypothetical

problems were constructed. In every case the curves arrived at by analysis of the data have practically coincided with the curves used in setting up the example. The method is therefore offered as a workable means of simultaneously obtaining curves showing the relation of one variable to several others, even where the shape of the curve is not known on *a priori* grounds.

Arithmetic of the successive approximation method. The method may best be illustrated by setting up a hypothetical case with given curvilinear relations, and then analyzing the relations by the approximation method to determine the curves. Table 1 shows 30 sets of observations of 3 variables highly correlated with each other. These are denoted by X_2, X_3, and X_4. Variable X_1 has been computed for each set of observations by the use of the equation:

$$X_1 = X_2 + \frac{20}{X_3} + 2\sqrt{X_4} - 5.$$

Assuming that we know nothing of the relation of X_1 to the other variables, let us proceed to work it out by the approximation method.

The raw data (Figure 2) show that X_1 is related to X_2, X_3, and X_4; but the correlations between the independent variables are so high that the true relations may be quite different from the apparent relations.

The first step is to determine the net linear relations by the usual multiple correlation methods. This gives the regression equation:

$$X'_1 = 2.48 + 1.0394X_2 - .2649X_3 + .3273X_4.$$

Computing the residual for each set of observations gives values as shown in Table 1. These residuals are next plotted from the net regression lines for each variable as shown in Figure 3. The net regression line on each chart is drawn so as to pass through the intersection of the means of the dependent variable and the independent variable, with a slope equal to the net regression of X_1 on the independent variable. The residuals are then plotted with the accompanying value of the independent variable as abscissa, as vertical deviations from the net regression line.

It is worth considering exactly what these plotted values mean. Taking deviations from the means, the regression line for X_1 on X_2 has the formula:

$y = b_2 x_2$ when the mean of x_1 is the zero ordinate.
(In this case, $y = 1.0394x_2$)

Plotting the residuals as deviation from this line with X_2 as abscissa, the ordinate of each dot becomes:

$$y = b_2 x_2 + e.$$

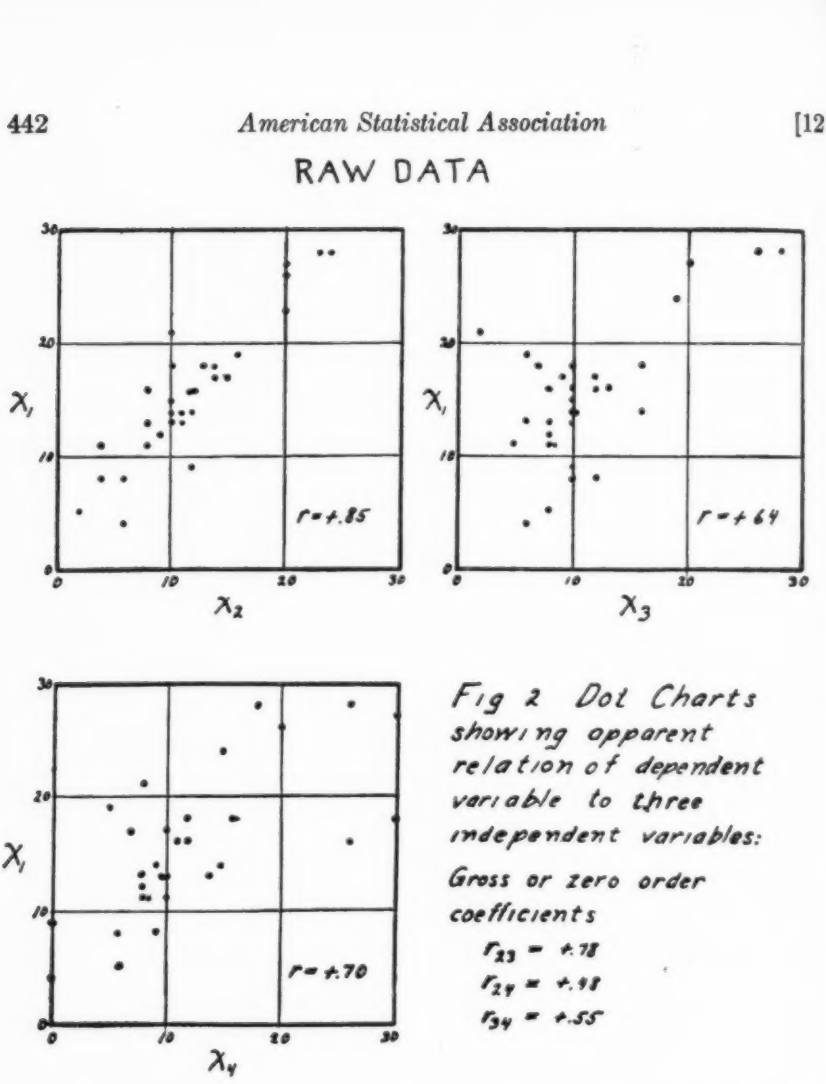


Fig 2 Dot Charts showing apparent relation of dependent variable to three independent variables:

Gross or zero order coefficients

$r_{23} = +.78$
$r_{24} = +.98$
$r_{34} = +.55$

Now, the value of e, in the regression equation in terms of deviations from the means (Eq. 1), is

$$e = x_1 - (b_2x_2 + b_3x_3 + b_4x_4).$$

Substituting this value for e in the preceding equation:

$$\begin{aligned} y &= b_2x_2 + x_1 - (b_2x_2 + b_3x_3 + b_4x_4) \\ &= x_1 - b_3x_3 - b_4x_4. \end{aligned}$$

[In this case, $y = x_1 - (-.2649x_3 + .3273x_4.)$]

That is, the dots plotted on the X_1X_2 chart have for ordinates the values of x_1 less linear corrections for the effects of deviations of accompanying values of X_3 and X_4 from their means.

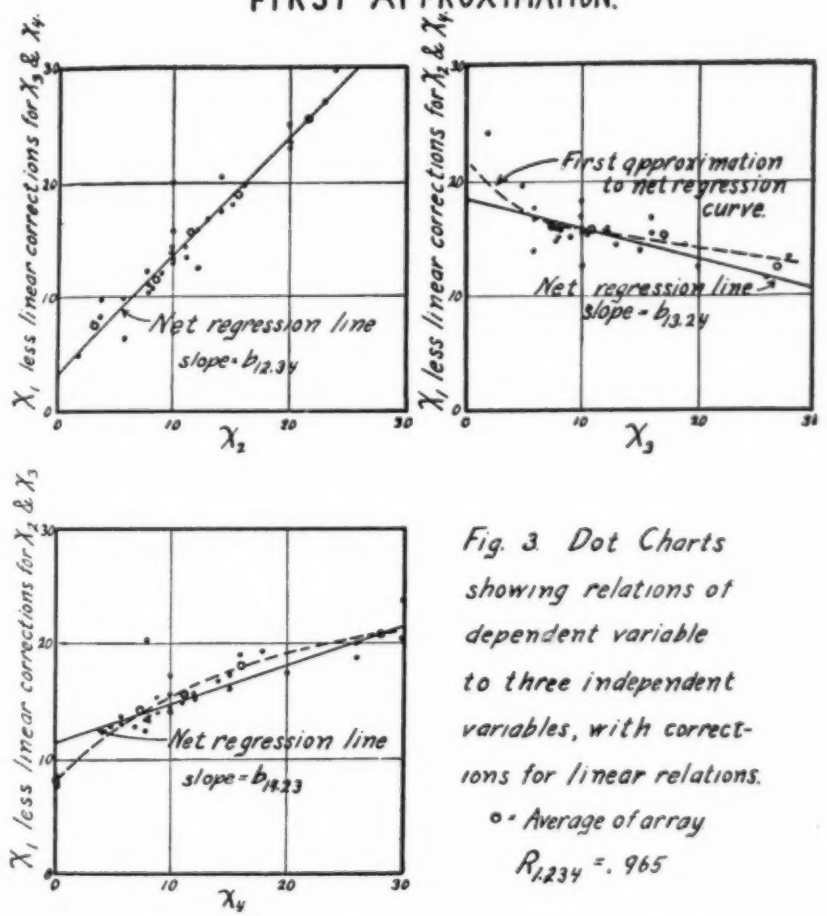


Fig. 3. Dot Charts showing relations of dependent variable to three independent variables, with corrections for linear relations.

o = Average of array

$R_{1.234} = .965$

Similar considerations apply to the X_1X_3 and X_1X_4 charts; the ordinates of the dots on the X_1X_3 chart are equal to $x_1 - b_2x_2 - b_4x_4$; on the X_1X_4 chart, to $x_1 - b_2x_2 - b_3x_3$.

Comparing Figures 3 with Figure 2, it is evident that this correction has much reduced the scatter. The relation of X_1 to each of the other variables is therefore much better defined in Figure 3.

Before drawing curves freehand as indicated by the dots, averages of the residuals are taken over arrays of the several independent variables to aid in drawing the curves. The arrays used are 0–4; 5–9, 10–14, etc. The averages of the residuals within each array are plotted as deviations from the regression line, with abscissa equal to the average of the values of the independent variable within that array.

The chart X_1X_2 has these averages falling alternately first above and then below the regression line, and no curve would give a better fit. For X_1X_3, all except one average (based on only one observation) indicate a curve. A freehand curve is fitted roughly to these points, not going as high, however, as the first dot, as the single observation is quite apt to be in error. The averages in X_1X_4 also strongly indicate a curve, and one is fitted freehand as shown in the chart.

The second residual is now computed by formula (2), using the values of the straight line for $f'(X_2)$, and reading the values for $f'(X_3)$ and $f'(X_4)$ directly from the curves of Figure 3. The constant K' should be found by correlating the sum of the values of $f'(X_2)$, $f'(X_3)$, $f'(X_4)$ for each observation with X_1; but for practical purposes this can be done by taking K' as a value which will make the sum of the residuals zero. That is,

$$K' = \frac{\Sigma\{X_1 - (f'X_2 + f'X_3 + f'X_4)\}}{n}.$$

The estimated value for X_1 thus obtained, and the second residuals, are shown in Table 1. By definition, the index of multiple correlation is equal to the standard deviation of the estimated values of X_1 divided by the standard deviation of the observed values.

$$\text{Now } X_1 = X'_1 + e'$$

and since the residuals are uncorrelated with the predicted values, X'_1

$$\sigma^2_{X_1} = \sigma^2_{X'_1} + \sigma^2_{e'}$$

and

$$\sigma^2_{X_1} - \sigma^2_{e'} = \sigma^2_{X'_1}$$

Hence

$$P^2 = \frac{\sigma^2_{X'_1}}{\sigma^2_{X_1}} = \frac{\sigma^2_{X_1} - \sigma^2_{e'}}{\sigma^2_{X_1}} = 1 - \frac{\sigma^2_{e'}}{\sigma^2_{X_1}}. \tag{4}$$

This last formula is exactly analogous to the formula showing the relation of the coefficient of multiple correlation to the standard deviation of the residuals from the linear estimates of X_1.

$$\sigma^2_e = \sigma^2_1(1 - R); \quad R^2 = 1 - \frac{\sigma^2_e}{\sigma^2_1}.$$

Using formula (4), $P = .979$ for the first approximation curves, a value significantly higher than that of R, which is .965.

Figure 4 shows these second residuals plotted as deviations from the first approximation curves. Taking averages of the residuals as before, and fitting curves to them by freehand smoothing, gives second approximation curves.

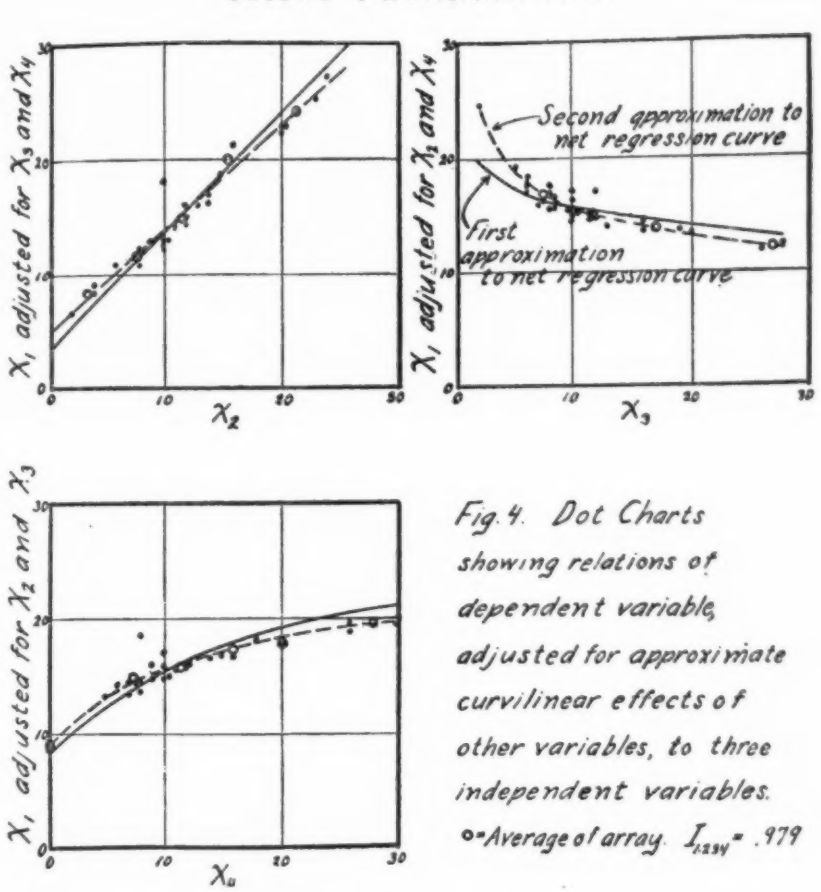


Fig. 4. Dot Charts showing relations of dependent variable, adjusted for approximate curvilinear effects of other variables, to three independent variables.
o-Average of array. $I_{123y} = .979$

Computing third residuals by formula (3) gives values as shown in Table 1. Computing the multiple correlation for these second approximation curves by (4), $P = .981$. Apparently the approximation has been carried nearly as far as is worth while.

A further correction to the curves is made as follows:

In each set of the original series of observations

$$X_1,\ X_2,\ X_3,\ X_4$$

for the dependent variables the values given by the regression curves are substituted. These values are given in Table 2. These new values are

$$f''(X_2),\ f''(X_3),\ f''(X_4)$$

the functions being the values read off the curves. Then by multiple correlation the X_1 values are correlated with these values. This gives the regression equation:

$$X_1 = -32.28 + 1.12f''(X_2) + .84f''(X_3) + 1.09f''(X_4)$$

with an index of multiple correlation of .994.

Apparently the general slope of the line for X_2 and the curve for X_4 were not steep enough, and the curve for X_3 was a little too steep. Multiplying the ordinates of the second approximation curves by these corrections gives third approximation curves. Figure 5 shows the third approximation curves together with the graphs of the functions used in constructing the example, plotted so that the values corresponding to the mean value of the independent variable coincide. Through the portion of the curves covered by the body of the data

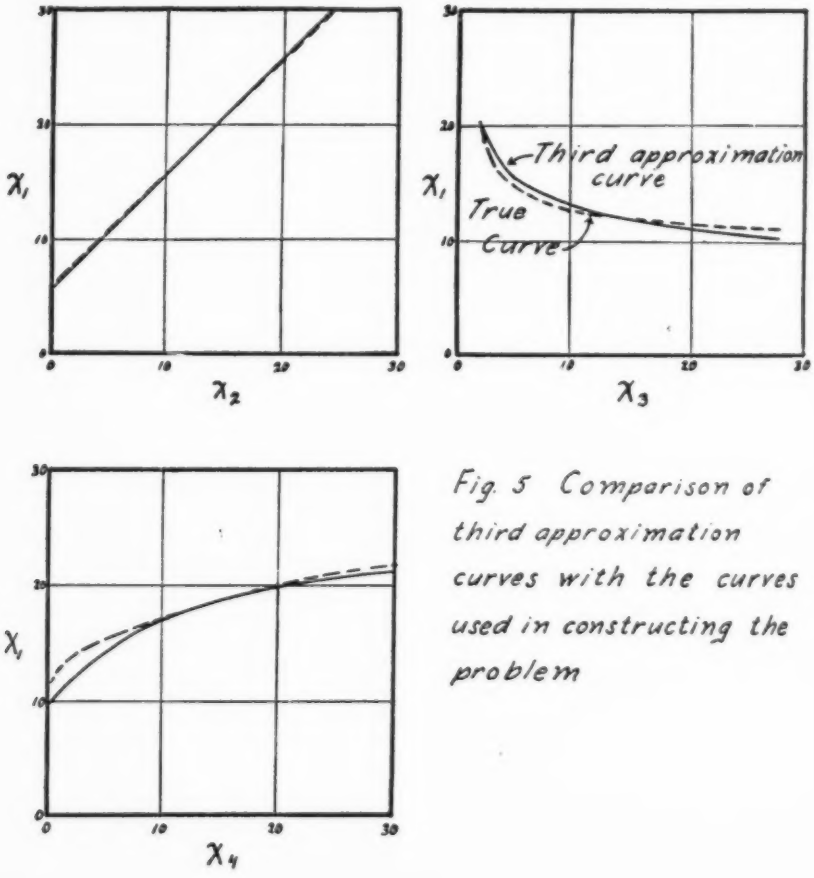


Fig. 5 Comparison of third approximation curves with the curves used in constructing the problem

the curves found by approximation agree fairly well with the true curves.

Computing third residuals from predictions made from the third approximation curves gives the values shown in the last column of Table 1. Figure 6 shows these third residuals plotted as deviations from the third approximation curves. It is apparent that if the process of approximation were continued still further even a closer approach to the true curves would be obtained. Thus the curvilinear regression of X_1 on X_3 indicated by the dots in Figure 6 approaches closer to the true curve as shown in Figure 5 than does the third approximation curve.

The number of approximations it will pay to carry a given problem through will depend upon the accuracy desired. As illustrated by this example, the best possible definition of the regressions from a given

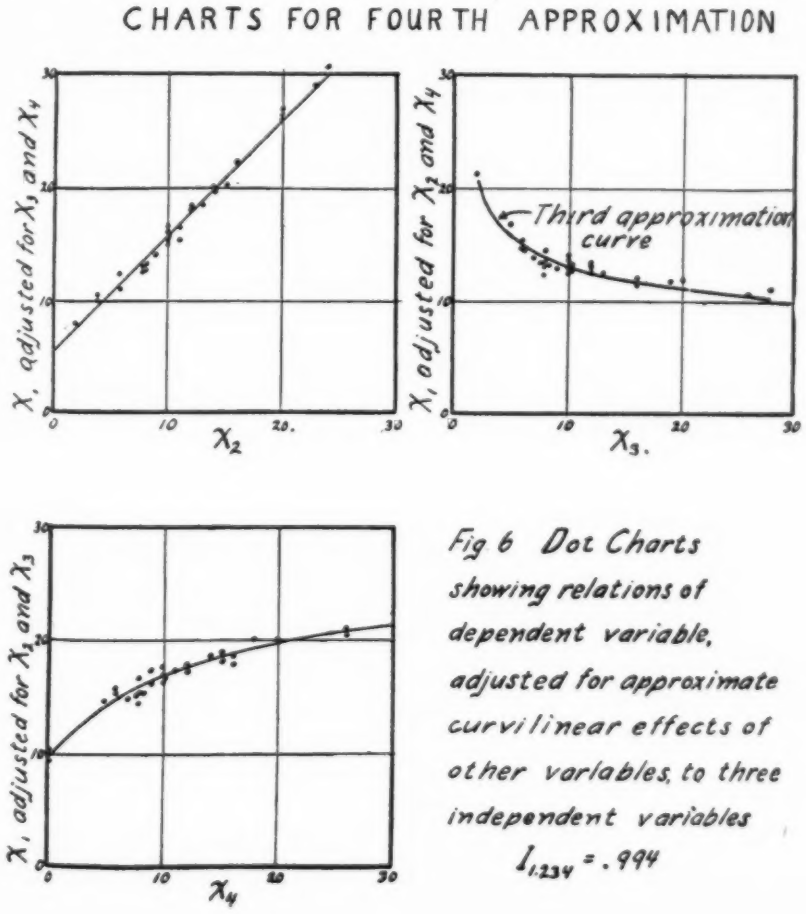


Fig 6 Dot Charts showing relations of dependent variable, adjusted for approximate curvilinear effects of other variables, to three independent variables

$I_{1.234} = .994$

set of data will be obtained when the final residuals, plotted as shown, indicate no further changes in any of the curves.

In concluding the discussion of the example, it is interesting to note that the computed values of X_1 were reduced to the nearest unit in setting up the problem. Only 12 of the last residuals are larger than 0.5, showing how closely the curvilinear regression equation estimated the values, in view of the failure to state the X_1 values more exactly.

In approximately determining net curvilinear regressions by this method, it is not necessary to make dot-charts, as in the example. Where there are sufficient observations the residuals are averaged as indicated, and the curves drawn on charts showing only these averages plotted from the net regression lines. By the use of a sufficiently large scale the curves can be read to any degree of accuracy desired.

The volume of work required by this method is not as large as it may appear. Once the residuals are computed for the linear net regressions the work of averaging and plotting residuals, determining new curves, and estimating new values is quite rapid. A complete analysis by this method takes about two to three times as long as the linear multiple correlation analysis of the same data.

Index of net correlation for approximation curves. The final multiple correlation determination of the best slope of each curve affords not only an exact measure of the curvilinear correlation of the several independent variables with the dependent variable, but also opens a way to obtain indexes of net (or partial) correlation, measuring the net correlation between the dependent variable and the function of each independent variable, with the curvilinear effects of all other variables held constant.

That is, when in each set of observations

$$X_1,\ X_2,\ X_3,\ X_4.$$

for the observed value of the dependent variables the values given by the regression curve are substituted, and then these values $f''(X_2)$, $f''(X_3)$, $f''(X_4)$, correlated with the observed value of X_1, then

$$R_{X_1.f''(X_2)f''(X_3)f''(X_4)} = P_{X_1.X_2X_3X_4.}$$

and similarly

$$r_{X_1f''(X_2).f''(X_3)f''(X_4)} = \rho_{X_1X_2.X_3X_4}$$

$$r_{X_1f''(X_3).f''(X_2)f''(X_4)} = \rho_{X_1X_3.X_2X_4}$$

and so on.[1]

Adequacy of the Approximation Method for Determining Net Curvilinear Regressions. Where all independent variables are entirely uncor-

[1] This method of determining the index of net correlation was suggested by Dr. Sewall Wright. The author wishes to take this opportunity of thanking him for his advice and help.

related, and enough observations are available, the curvilinear relation of any variable to the dependent variable may be determined by simple averages, since the effects of other variables will cancel out. Curves fitted to these averages, either analytically or graphically, will serve to define the relation.

However, when the independent variables are intercorrelated to a greater or less extent, averages alone will not give true curvilinear relations, as the effect of correlated variables will be reflected in the averages.

The elimination of the net linear effects of correlated variables, as shown in the first step in the illustrative case, brings the averages somewhat nearer the true curve. While the original averages include all the effect of intercorrelation among the variables, the averages after this first correction are affected only by correlated *deviations from linearity*. That is, if the curve of X_3 tends to deviate positively from its net regression at a value correlated with a value of X_4 which has a similar positive deviation from its net regression, the combined effect of the two positive deviations would show in the corrected average, first for X_3 and then for X_4. It is evident, however, that this effect will ordinarily be much slighter than the total correlation. Comparison of Figure 3 with Figure 2 shows how elimination of only the linear regressions improved the definition of the curves. In ordinary practice, it is probable that such intercorrelations will not be sufficient to prevent the true curves from roughly showing up at this point. Once the curvilinear relations begin to appear, even though imperfectly, each successive approximation reduces the deviation of the corrected averages and brings more exact definition to the curves until the exact shape can be determined with a fair degree of assurance.

Probable error of the curves. The use of the method of approximation does not change the situation with regard to the necessity for a considerable number of cases if reliance is to be placed in the conclusions. Spuriously high correlation could be obtained by drawing curves with many inflections so as to pass through as many of the individual observations or averages as possible. In most cases, however, some reasonably smooth curve would seem to be indicated, and this should be borne in mind in smoothing the line of average residuals.

Where sufficient observations are available so that each average represents a number of observations, there would be reasonable accuracy in the definition of the curve. In determining the curves by this method the possibility of error must be thought of in regard to the various parts of the curve, rather than for the entire length. Thus with a normal frequency distribution of an independent variable in a

sample of say 100 observations, the shape of the curve found along the central portions would probably be quite accurate, while there would be more chance of error at the extremities of the curve, determined by only a very few observations. An inflection at the extreme end of a curve, based on only one or two observations, would be quite questionable, while a similar inflection based on a large number of observations could be quite confidently accepted.

It might be possible to apply the theory of probability to determine the probable reliability of various portions of the curve; but with the present roughness of method, it would seem hardly worth while.

III

ARITHMETIC EXAMPLES OF THE MULTIPLE CURVILINEAR APPROXIMATION METHOD

The approximation method of determining the regressions for multiple curvilinear relations has been tried in a number of studies. Two will be cited here as examples of the possibilities of the method.

Factors determining the rate of fattening for steers were studied from farm records for 67 droves representing about 2,000 head of cattle. Starting with a linear multiple correlation of less than .75, curvilinear analysis gave $P = .820$ for the multiple correlation of daily gain in weight with five factors. The curves showed that as the daily feed of corn or of hay was increased the increase in gain per pound of corn became less and less—an actual measurement of the curves of diminishing returns. Two other curves showed the relation of the weight of the steers and of the length of the feeding period to the economy of gain—each curve being "net" of the effects due to the three other variables. The fifth factor—protein content—gave only a linear regression.[1]

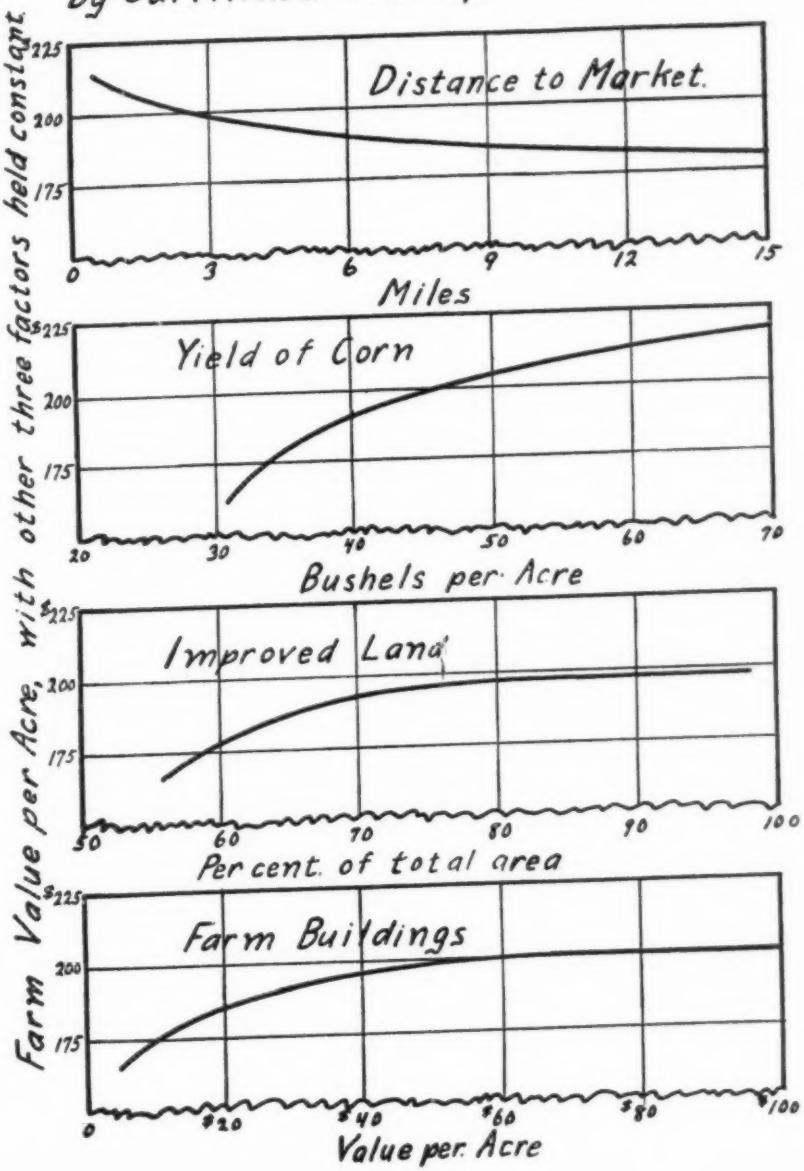
In another study[2] covering 345 Indiana farms, four factors gave a multiple correlation of $R = .58$ with farm value per acre. Determining the curves raised the multiple correlation to $P = .68$. The relation shown by the curves in this case were particularly significant. The net regression curves are shown in Figure 7.

As would be expected, the farther the farms were from their market town, the less they were worth. The effect is diminishing, however; an increase in distance from 6 to 9 miles lowered the value only half as much as an increase from 3 to 6 miles.

[1] These curves are shown on pages 19 and 20 of *U. S. Dept. of Agriculture Bulletin* 1277, "Input as Related to Output in Farm Organization and Cost-of-Production Studies," by H. R. Tolley, J. D. Black, and M. J. B. Ezekiel.

[2] The author is indebted to Mr. C. R. Chambers, formerly of the Bureau of Agricultural Economics, for furnishing this illustration.

Fig. 7.
Net Relations of Four Factors to the Value of Indiana Farms: determined by Curvilinear Multiple Correlation.



The fertility of the soil is necessarily one of the prime factors affecting the value of a farm. Using the yield of corn as an index of soil fertility, the curve shows that the higher the yield the less is the effect of a further increase. An increase from 50 to 60 bushels raised the farm value only half as much as an equal increase from 40 to 50 bushels.

Since crop production is important in this section, unless four-fifths of the farm area was improved the values were lowered. But since a certain proportion of pasture land was needed for live stock, no further increase in value was obtained when the proportion of improved land was higher than 80 per cent.

It is generally recognized that the farm buildings constitute a considerable part of farm values. The net regression curve shows that up to $20 worth per acre the buildings added as much to the farm value as they cost, or more. Beyond that point the increase in farm value with increase in the value of the buildings became less, until beyond

TABLE I

DATA OF ARITHMETIC EXAMPLE

$$(X_1 = X_2 + \frac{20}{X_3} + 2\sqrt{X_4} - 5)$$

Item No.	Raw Data				Estimated Values of X_1				Residuals			
	X_2	X_3	X_4	X_1	Linear regression	First approx. curves	Second approx. curves	Third approx. curves	e	e'	e''	e'''
1	11	10	9	14	14.2	14.0	14.4	14.2	−0.2	0.0	−0.4	−0.2
2	20	19	15	24	23.0	25.0	22.1	23.5	+1.0	−1.0	+1.9	+0.5
3	6	6	0	4	7.1	3.3	5.7	4.2	−3.1	+0.7	−1.7	−0.2
4	6	12	6	8	7.7	6.6	7.6	7.0	+0.3	+1.4	+0.4	+1.0
5	8	8	26	16	17.2	16.8	16.5	16.3	−1.2	−0.8	−0.5	−0.3
6	9	8	8	12	12.4	11.5	13.1	12.6	−0.4	+0.5	−1.1	−0.6
7	11	8	8	13	14.4	13.5	15.0	14.7	−1.4	−0.5	−2.0	−1.7
8	14	16	16	18	15.7	19.5	17.4	18.1	+2.3	−1.5	+0.6	−0.1
9	12	10	0	9	12.4	8.5	9.1	8.5	−3.4	+0.5	−0.1	+0.5
10	8	8	8	11	11.3	10.4	12.2	11.6	−0.3	+0.6	−1.2	−0.6
11	4	5	10	11	8.7	9.1	11.2	9.9	+2.3	+1.9	−0.2	+1.1
12	23	26	26	28	28.0	29.7	25.8	27.9	0.0	−1.7	+2.2	+0.1
13	14	12	10	17	17.2	17.7	16.5	16.9	−0.2	−0.7	+0.5	+0.1
14	10	16	14	14	13.3	14.6	13.4	13.6	+0.7	−0.6	+0.6	+0.4
15	10	10	15	15	15.2	15.9	15.4	15.3	−0.2	−0.9	−0.4	−0.3
16	20	13	20	26	26.5	27.3	24.5	25.8	−0.5	−1.3	+1.5	+0.2
17	12	12	12	16	15.7	16.6	16.5	15.7	+0.3	−0.6	−0.5	+0.3
18	10	2	8	21	15.0	16.3	21.9	20.2	+6.0	+4.7	−0.9	+0.8
19	16	6	5	19	19.1	17.6	18.7	18.7	−0.1	+1.4	+0.3	+0.3
20	20	20	30	27	27.9	28.0	24.4	26.0	−0.9	−1.0	+2.6	+1.0
21	10	10	10	13	13.6	13.7	13.8	13.5	−0.6	−0.7	−0.8	−0.5
22	2	8	6	5	5.2	3.7	5.7	4.3	−0.2	+1.3	−0.7	+0.7
23	8	8	8	11	11.3	10.4	12.2	11.6	−0.3	+0.6	−1.2	−0.6
24	12	10	11	16	16.0	16.3	15.9	15.9	0.0	−0.3	+0.1	+0.1
25	13	7	12	18	18.0	18.5	18.2	18.3	0.0	−0.5	−0.2	−0.3
26	15	9	7	17	18.0	17.0	17.5	17.8	−1.0	0.0	−0.5	−0.8
27	24	28	18	28	26.0	28.6	24.8	27.0	+2.0	−0.6	+3.2	+1.0
28	10	10	30	18	20.2	19.1	17.9	18.0	+2.2	−1.1	+0.1	0.0
29	4	10	9	8	7.0	6.8	7.8	7.0	+1.0	+1.2	+0.2	+1.0
30	8	6	10	13	12.2	12.7	14.0	13.2	+0.8	+0.3	−1.0	−0.2
Mean	11.67	11.10	12.23	15.67								
σ				6.28					1.65	1.28	1.21	0.67

$60 worth of buildings per acre, further increase had no effect upon the farm value.

Each of these curves illustrates and substantiates conclusions reached by theoretical economic analysis. Equally important, they provide definite quantitative statements of the relationships. The method of multiple correlation, and this derivative from it, the method of curvilinear multiple correlation, enable us to use the favorite tool of the economist, *caeteris paribus*, in the analysis of actual happenings equally as well as in the intricacies of theoretical reasoning.

TABLE II

DATA FOR FINAL CORRELATION FIT

Item No.	Original Values			Curvilinear Equivalents of			Original X_1
	X_2	X_3	X_4	X_2	X_3	X_4	
				$f''(X_2)$	$f''(X_3)$	$f''(X_4)$	
1	11	10	9	14.9	15.9	15.2	14
2	20	19	15	23.0	13.6	17.1	24
3	6	6	0	10.3	18.0	9.0	4
4	6	12	6	10.3	15.1	13.8	8
5	8	8	26	12.1	16.8	19.2	16
6	9	8	8	13.0	16.8	14.9	12
7	11	8	8	14.9	16.8	14.9	13
8	14	16	16	18.5	14.2	17.3	18
9	12	10	0	15.8	15.9	9.0	9
10	8	8	8	12.1	16.8	14.9	11
11	4	5	10	8.3	19.0	15.5	11
12	23	26	26	25.7	12.5	19.2	28
13	14	12	10	17.5	15.1	15.5	17
14	10	16	14	14.0	14.2	16.8	14
15	10	10	15	14.0	15.9	17.1	15
16	20	13	20	23.0	14.9	18.2	26
17	12	12	12	16.8	15.1	16.2	16
18	10	2	8	14.0	24.6	14.9	21
19	16	6	5	19.2	18.0	13.1	19
20	20	20	30	23.0	13.4	19.6	27
21	10	10	10	14.0	15.9	15.5	13
22	2	8	6	6.7	16.8	13.8	5
23	8	8	8	12.1	16.8	14.9	11
24	12	10	11	15.8	15.9	15.8	16
25	13	7	12	16.6	17.0	16.2	18
26	15	9	7	18.4	16.3	14.4	17
27	24	28	18	26.5	12.1	17.8	28
28	10	10	30	14.0	15.9	19.6	18
29	4	10	9	8.3	15.9	15.2	8
30	8	6	10	12.1	18.0	15.5	13

A NEW ZEALAND STUDY IN NATALITY

By Edward P. Neale, *Victoria University College, Wellington, N. Z.*

Throughout the whole of New Zealand compulsory registration of births, deaths, and marriages is in force; and there is good reason for believing that the statistics are very accurate.[1] They are moreover remarkably full. This article is an attempt to present the more important uniformities which can be brought out by an analysis of the statistics of births. The statistics do not include the Maori population.

FACTORS AFFECTING FECUNDITY

As an index of fecundity[2] legitimate children born alive may be considered in relation to the numbers of married women of childbearing age returned at the nearest census;[3] illegitimate children born alive in relation to the unmarried (*i. e.* widowed, divorced, and never married) women of childbearing age at the nearest census; or the two results may be combined in one figure by considering total children born alive in relation to total women of childbearing age at the nearest census. Childbearing age I have considered to be ages 15 to 44 inclusive at the last birthday; this is sufficiently approximate for most practical purposes. It is certain that maternity has different frequencies for (a) the widowed; (b) the divorced; and (c) the never married: no statistic is, however, available in New Zealand whereby such considerations could be allowed for. The figures assume moreover that no illegitimate children are born to married women: this is not absolutely correct, but the error introduced by this assumption cannot be large, at least so long as the term "illegitimate" is confined to children actually registered as such, as it needs must be for present purposes. The results are as shown in Table I.

The secular downward trend of the birth-rate, which is so well known a characteristic of countries with a Western civilization, will be seen to be exhibited also by the New Zealand statistics.

Table I is the best measure of fecundity derivable from the avail-

[1] See hereon the present writer's article in the *Journal of the Royal Statistical Society*, March, 1923, and the references there quoted.

[2] With regard to the terms "fertility" and "fecundity," whose connotation has not yet been standardized, I follow the usage of Knibbs in his *Mathematical Theory of Population*, Appendix A to Vol. I of the Results of the 1911 Australian Census rather than the usage of Carr-Saunders in his *Population Problem*, etc.

[3] From 1881 onwards a census has been held quinquennially in New Zealand. Prior to 1881, three census enumerations took place in each decade.

TABLE I

INDEXES OF FECUNDITY, 1876-9 TO 1914-18

Average of years	Children born alive per 1,000 women of reproductive age	Legitimate children born alive per 1,000 wives of reproductive age	Illegitimate children born alive per 1,000 unmarried women of reproductive age
1876-79	214.4	330.0	13.3
1880-83	198.4	322.8	13.5
1884-88	164.3	299.4	10.9
1889-93	138.7	278.5	9.2
1894-98	118.0	256.1	9.0
1899-03	110.5	243.7	8.8
1904-08	114.5	236.7	9.6
1909-13	111.6	215.3	9.6
1914-18	103.9	188.0	9.4

able statistics, but it makes no allowance for variations from time to time in the age constitution of the different groups of women into which the female population of reproductive age has been for purposes of the above table divided. From 1915 however statistics are available which do permit of allowance being made for differences in the age constitutions of these groups. The average children born alive for the years 1915–1917 considered in conjunction with the 1916 Census results yields the following results:

TABLE II

FECUNDITY ACCORDING TO AGE-GROUPS, 1915-1917

Age of mother last birthday	Children born alive per year to women in age group per 1,000 women belonging to group	Legitimate children born alive per year to wives in age group per 1,000 wives belonging to group	Illegitimate children born alive per year to women in age group per 1,000 unmarried women belonging to that group
15-19	18.0	520.7	6.2
20-24	118.4	369.1	13.2
25-29	173.7	275.7	11.5
30-34	150.8	197.2	9.4
35-39	104.8	132.0	7.5
40-44	44.2	55.4	3.4
15-44	105.7	191.4	9.1

These figures relate of course to the frequency of maternity according to the age of mothers, and reveal a maximum legitimate fecundity at the earliest age-group and a maximum illegitimate fecundity at the age-group 20 and under 25. So far it has been assumed that the ages and conjugal conditions of women alone determine the average frequency of maternity. It is probably the case, however, that the following factors are not without their influence on fecundity: (a) the age of the associated male; (b) the number of previous pregnancies of the mother; (c) the number of previous confinements of the mother;

(d) the number of previous issue of the mother; (e) the age at which childbearing initiated; and (f) the duration of married life. It should be noted that factors (b), (c), and (d) are not necessarily identical, nor are factors (e) and (f).

Let us consider first the influence on the fecundity of wives exerted by the husband's age. The 1916 Census affords statistics of the relative ages of husbands and wives returned on the same schedule. The husbands of rather fewer than one-sixth of the married women returned at this census were not included on the same schedule as their wives, and I have allocated these doubtful cases to the various age-groups on the assumption that the "absent" husbands of women of a given age were distributed among the different age-groups in the same proportion as the "present" husbands of women of the same age. The children born alive during the triennium 1915–1917 were considered according to the respective ages of their parents, these figures being divided by 3 in order to arrive at the average number of children born per year to parents in each conjunction. The fecundity per 100 couples was then arrived at by dividing the last results by the number of hundreds of couples ascertained to belong to the corresponding conjunctions. All cases where 20 or fewer children were born to a conjunction and all cases where fewer than 22 couples belonging to a conjunction were returned as living together at the date of the census are excluded from consideration; hence the blanks in the table:

TABLE III

CHILDREN BORN ALIVE ANNUALLY, 1915–1917, PER 100 COUPLES CLASSIFIED ACCORDING TO THE RESPECTIVE AGES OF THE SPOUSES

Mothers' ages in years last birthday	Fathers' ages in years last birthday										Total
	Under 21	21–24	25–29	30–34	35–39	40–44	45–49	50–54	55–59	65 and over	
Under 20	67	55	47	53	..	..	..	..	..	..	52
20–24	54	42	37	34	36	37	32	..	..	..	37
25–29	..	33	30	26	25	24	23	28	23	..	28
30–34	..	24	27	21	19	18	16	16	16	..	20
35–39	..	..	24	17	14	12	12	11	9	8	13
40–44	..	..	19	13	8	6	5	5	4	..	6
Under 45	60	40	33	25	17	12	10	9	8	5	19

It will be obvious from the foregoing table that fecundity decreases not only with an increase in the wife's age but with an increase in that of the husband also. The few anomalies that exist are probably due to:

1. The fact that fecundity suffered during the period under review by the absence on war service of husbands belonging to young rather than to old age-groups.

2. The error involved in the assumption that the age distribution of "absent" husbands was identical with that of husbands returned in the 1916 Census along with their wives.

3. Fortuitous synchronizings (or the opposite) of births in particular conjunctions during the period reviewed. Increasing the number of years covered by the investigation would tend to eliminate such influences, but unfortunately the necessary data are not extant.

It will also be interesting to consider to what extent the fecundity of wives varies with the number of issue that have previously been born to them. (Data as to the number of previous pregnancies and the number of previous confinements are not extant.) The following table has been compiled on similar lines, *mutatis mutandis*, to that last quoted. Conjunctions where fewer than 20 children were born alive are again rejected on the ground that paucity of instances is liable to lead to a high relative error owing to the disproportionate influence exerted by fortuitous coincidences on the values of the coefficients deduced.

TABLE IV

CHILDREN BORN ALIVE ANNUALLY, 1915-1917, PER 100 WIVES CLASSIFIED ACCORDING TO AGE AND TO NUMBER OF PREVIOUS ISSUE

Mothers' ages in years last birthday	Number of previous issue born													Total
	0	1	2	3	4	5	6	7	8	9	10	11	12	
Under 20	84	19	..	..	..	..	..	..	..	..	..	..	..	52
20–24	61	29	22	18	14	..	..	..	..	..	..	..	..	37
25–29	40	27	23	21	21	22	22	23	..	..	..	..	..	28
30–34	21	21	18	17	20	21	22	24	21	24	..	..	..	20
35–39	9	13	11	12	13	16	18	22	18	22	22	22	..	13
40–44	2	4	4	4	5	6	8	9	11	13	13	16	16	6
Under 45	29	21	16	14	14	15	16	18	16	19	20	21	24	19

The proportion of married women bearing their first child to total married women without previous issue falls gradually with increasing age. This is what might have been expected *a priori*, for the class "married women with no previous issue" contains at early age-groups an almost overwhelming proportion of "newly-weds." As age increases this class is to an ever-increasing extent made up of women physiologically sterile and of women who intend to avoid bearing children. In each of the earlier age-groups the highest percentage of births is among those women who have had no previous issue, and fecundity diminishes with an increase in the number of previous issue until eventually a minimum is reached, after which fecundity appears

to increase with an increase in the number of previous issue. With increasing age this minimum is to be found at an ever-diminishing number of previous issue, until at the highest age-groups the minimum fecundity is to be found among women with no previous issue. Actually, when a woman has borne a certain number of children, she appears on the average to begin to have them more frequently thereafter. It does not follow by any means, however, that this circumstance is wholly, if at all, to be accounted for by physiological considerations. The real implication of the figures appears rather to be this. The bulk of present day married women intend to have but few children, and the interval between successive confinements is with them a lengthy one. Such women form a large proportion of the total married women with one or two previous issue; therefore the frequency of maternity is comparatively low for women in general with such a number of previous issue. Some few women on the other hand have no intention of putting a limit to the number of their offspring, and with them the interval between successive confinements is short; in other words maternity is frequent. Only such women qualify for inclusion in the classes "women with 4, 5, 6, and higher numbers of previous issue"; hence the high frequency of maternity in these classes. In the lower classes these women are comparatively few, and the high frequency of maternity among them is swamped by the low frequency among the great bulk of the women belonging to those classes. The speeding-up of childbearing in the higher classes is therefore more apparent than real.

Data are unfortunately not available for a satisfactory investigation of fecundity according to the ages of mothers in relation to the durations of their marriages.

SEXES OF CHILDREN BORN

The number of males per 100 females amongst children born alive is shown in Table V.

TABLE V

Year	Per cent	Year	Per cent	Year	Per cent	Year	Per cent
1866	102.4	1880	104.7	1894	104.6	1908	106.3
1867	106.6	1881	104.9	1895	104.9	1909	103.7
1868	103.0	1882	104.5	1896	104.5	1910	107.2
1869	100.7	1883	105.2	1897	105.1	1911	105.5
1870	104.0	1884	104.3	1898	102.9	1912	103.6
1871	104.4	1885	103.6	1899	106.7	1913	106.9
1872	104.3	1886	104.7	1900	107.1	1914	105.3
1873	103.5	1887	103.3	1901	104.5	1915	107.3
1874	106.8	1888	104.1	1902	106.5	1916	106.0
1875	107.8	1889	106.4	1903	105.7	1917	103.0
1876	106.0	1890	103.4	1904	106.9	1918	103.0
1877	104.6	1891	105.4	1905	104.6	1919	105.8
1878	101.6	1892	103.7	1906	104.6	1920	106.5
1879	105.9	1893	104.9	1907	104.7	1921	104.2

The secular trend appears to be in the direction of an increase in the proportion of males born. There is no evidence in the figures quoted however that there has been any tendency for the proportion to increase in connection with the war. Such a phenomenon has, however, been frequently observed in connection with European belligerents; and Franco Savorgnan, in a recent article in *Metron*,[1] ascribes it to an increased interval between intercourse owing to prolonged absences at the front on the part of procreators. Owing to New Zealand being so far from the storm centre short-leave home from the front was not feasible, so that the absence of an increase during the war period of the New Zealand proportion of male to female births is evidence in favour rather than in contradiction of Savorgnan's thesis.

For first births the proportion of males would appear to be much higher in New Zealand than for births in general. Figures on this point are available from 1919, and are shown in Table VI.

TABLE VI

Year	Males born alive where there has been no previous issue	Females born alive where there has been no previous issue	Males per 100 females
1919	2,851	2,634	108.2
1920	4,925	4,419	111.5
1921	4,711	4,384	107.5
1922	4,463	4,177	106.9

The secular decline in the birth-rate has meant smaller families, and therefore a higher proportion of first births. The fact that the masculinity of first births is greater than that of births in general taken in conjunction with this fact accounts to some degree for the diminishing proportion of girl babies, but whether or not it fully accounts for it the available figures do not show.

It might be suggested that the greater masculinity of first births does not depend on the order of confinement as such, but on the greater youth of the mothers. Statistics used to shed light on masculinity in relation to the ages of mothers should moreover be homogeneous as regards order of confinement in order that conclusive results may be arrived at. The figures in Table VII relate to first births only during the years 1919 to 1922 inclusive.

The figures are too small for any very great weight to attach to them, but the proportion of males certainly seems to be at a maximum among the children of mothers under 20. The masculinity of first births among older mothers would appear to be fairly uniform for all ages.

[1] Sept., 1921, pp. 137 ff.

TABLE VII

Age of mother last birthday	Males born	Females born	Males per 100 females
Under 20	1,041	920	113
20–24	6,321	5,943	106
25–29	5,726	5,179	111
30–34	2,625	2,414	109
35–39	980	915	107
40 and over	257	243	106
Totals	16,950	15,614	109

The fact that masculinity is apparently higher for first than for later births at all ages of mothers is further evidence in support of Savorgnan's thesis that masculinity varies inversely with the frequency of intercourse. First-born children are, as we shall see in the sequel, conceived for the most part just before or just after marriage, and this is especially true of the offspring of mothers under 20, among whom the masculinity of the children born is at a maximum. In such cases it is reasonable to suppose the frequency of intercourse will usually be much less just prior to the child's conception than in the case of children born to couples who have been married some time and are regularly cohabiting. It is to the latter class of couples that second and subsequent children are born, and among these masculinity is relatively low. Among 7,124 families considered during 1919–1921 by the New Zealand Census and Statistics Office the number of males per 100 females born alive was 112 for first-births, 104 for second births, 104 for third births, and 103 for fourth births.

No data are available as to variations with the ages of fathers in the masculinity of children born.

ILLEGITIMACY AND "PREJUDICED" NUPTIAL BIRTHS

Statistics of illegitimate births are available as far back as 1873, but there is good reason for believing that many illegitimate births were registered as legitimate prior to 1876. In that year a new Registration Act came into force, requiring the date and place of marriage of the parents of a child born to be entered at registration, and rendering it impossible for future mothers of illegitimate children to have them registered as the children of fictitious "husbands." Illegitimacy is perhaps, however, still very slightly understated in the statistics, as the illegitimate issue of married women are in all probability often registered in the name of the mother's husband; it may also sometimes be the case that the illegitimate offspring of a young single girl is officially registered as the legitimate issue of the woman who is in fact

its maternal grandmother. Until 1920 the illegitimate children of divorced women were probably in most instances registered in the names of the former husbands, whether in fact such persons were or were not the actual fathers; but by the passing of the Marriage Amendment Act, 1920, the Registrar-General is enjoined to enter particulars of all divorces granted on the relevant marriage records, thus providing a check on this practice.

The ratio of registrations of illegitimate children born alive per 100 total children born alive rose consistently from 2.23 in 1876 to 4.50 in 1895, since when it has remained remarkably constant, never exceeding 4.76 and never falling below 4.09. (These facts should, however, be considered in the light of Table I.) In the years 1916 to 1918 inclusive female illegitimates born alive actually exceeded in numbers male illegitimates born alive, this being at direct variance with what holds of births in general. During the years 1892 to 1920 inclusive 102.6 illegitimate males were born alive for every 100 illegitimate females, showing a much lower masculinity for illegitimate births than for births in general. This may be accounted for by the extraordinarily high masculinity of stillbirths, taken in conjunction with the comparatively high proportion of illegitimate stillbirths; of which more anon.

Closely allied to the matter of illegitimacy by reason of the light it sheds on sexual morality (or the lack of it) is the relation of first births to duration of marriages. Knibbs, at pp. 276 ff. of his *Mathematical Theory of Population*, while giving due weight to the occurrence of live births only 210 days after insemination, yet shows quite conclusively that the assumption is valid for most practical purposes that births occurring less than nine months after marriage are the result of antenuptial, births occurring more than nine months after marriage, of postnuptial, insemination. In any individual case this assumption is not valid, but in the aggregate it is, since negative divergences from the normal period almost exactly counterbalance positive divergences.

Figures for first living births in relation to duration of marriage are available each year from 1913 onwards. In Table VIII are shown first births, 1913 to 1920 inclusive, in relation to the ages of mothers and the duration of their marriages (if any). Of 425 first children born during these years to mothers under 17 on the birthday preceding the date of the child's birth 334 were illegitimate and 80 were born within nine months of marriage, and therefore apparently "prejudiced" by antenuptial insemination of their mothers, while only 11 were born in wedlock nine months or more after their mothers' marriages and apparently not therefore prejudiced in the same way.

TABLE VIII

Mothers' ages in years last birthday	Total of illegitimate births plus legitimate first births occurring within nine months of marriage	Total of illegitimate births plus all legitimate first births	Percentage of "prejudiced" cases to sum of illegitimates and legitimate firsts
13	3	3	100
14	26	26	100
15	107	107	100
16	278	289	96
17	792	852	93
18	1,572	1,831	86
19	2,201	2,947	75
20	2,422	3,762	65
21	2,652	4,725	56
22	2,503	5,802	43
23	2,142	5,767	37
24	1,735	5,561	31
25–29	5,109	22,394	23
30–34	1,985	10,942	18
35–39	964	4,780	20
40–44	318	1,225	26
45 and over	20	102	20
Total	24,830*	71,134*	35

* Includes 19 cases unspecified as to age.

The suggestion is that no marriages were other than "compulsory" which resulted in the birth of a child to a mother under 16. The proportion of "prejudiced" cases falls gradually with increasing age for at least two reasons: (a) increasing discretion; and (b) increasing knowledge of so-called neo-Malthusian practices. The implication of the figures is that of 100 women who became mothers for the first time under 21 years of age 76 either bore illegitimate children or married in haste for respectability's sake. More children are born in the seventh month after marriage than in any other month before the tenth: this may be ascribed to the fact that from two to three months will usually elapse between insemination and the date when a marriage can be arranged for after the fact that conception has occurred has been definitely established.

MULTIPLE BIRTHS

The percentage of multiple confinements per 100 total confinements annually from 1874 is given in Table IX.

TABLE IX

Year	Per cent	Year	Per cent	Year	Per cent	Year	Per cent	Year	Per cent
1874	1.19	1884	.97	1894	.97	1904	1.07	1914	1.27
1875	1.12	1885	.83	1895	1.08	1905	1.05	1915	1.10
1876	1.06	1886	.86	1896	.99	1906	.90	1916	1.15
1877	1.14	1887	1.00	1897	.98	1907	.98	1917	1.12
1878	.82	1888	1.03	1898	1.04	1908	1.11	1918	1.16
1879	.82	1889	.83	1899	.95	1909	1.01	1919	1.15
1880	.89	1890	1.06	1900	.99	1910	1.13	1920	1.24
1881	1.04	1891	1.00	1901	.95	1911	1.16	1921	1.25
1882	.95	1892	.98	1902	1.09	1912	1.22		
1883	.93	1893	1.05	1903	1.03	1913	1.14		

The nature of the fluctuations is not very clear, but there is clear evidence of an increasing tendency in the ratio of multiple to total confinements.

The ratio of multiple to total confinements might conceivably vary with one or more of the following factors: (a) the age of the mother; (b) the age of the associated male; (c) duration of the marriage (? age at which childbearing initiated); or (d) the number of previous issue (? pregnancies; ? confinements). Data are not available in New Zealand for investigating the phenomenon of multiple births in relation to the last two factors enumerated; but for 1913 and subsequent years it is possible to investigate the relative frequency of multiple births according to the ages of the parents. The following figures have been compiled by arranging (a) the total number of all confinements; and (b) the total number of multiple confinements, according to the ages of the parents, and obtaining quotients from corresponding items. (Cases where twins would have been registered had not one child been stillborn have been counted as single confinements for purposes of this table, not that such a practice is in itself desirable, but because such a proceeding is necessitated by the manner of presentation of the official statistics.) To eliminate the effects of fortuitous occurrences of multiple confinements in conjunctions where confinements are infrequent I do not quote data for conjunctions in which fewer than 1,000 confinements occurred during the eight years considered.

TABLE X

NUMBER OF MULTIPLE CONFINEMENTS PER 100 TOTAL CONFINEMENTS CLASSIFIED ACCORDING TO THE AGES OF SPOUSES, 1913–1920

Mothers' ages in years last birthday	Fathers' ages in years last birthday									Total Legitimate	Illegitimate	Total
	Under 21	21–24	25–29	30–34	35–39	40–44	45–49	50–54	55–64			
Under 20	..	.6	.2	..	..	..	..	..	..	.3	.3	.3
20–24	..	.7	.8	.7	.9	1.2	..	..	..	.8	.9	.8
25–29	..	1.2	.9	1.1	1.1	1.0	..	..	..	1.0	1.1	1.0
30–34	..	..	1.5	1.4	1.3	1.4	1.5	..	..	1.4	1.7	1.4
35–39	..	..	..	1.8	1.7	1.6	1.8	1.7	..	1.7	..	1.7
40–44	..	..	..	..	1.1	1.2	1.2	1.0	..	1.2	..	1.1
Totals	.7	.8	.9	1.2	1.3	1.4	1.5	1.2	.8	1.2	.9	1.2

The message of these figures is unmistakable. Multiple confinements are at a maximum when the mother is 35 and under 40 and the father 45 and under 50. The gradations on either side of the maximum are remarkably uniform.

Among illegitimate confinements the ratio of multiple to total

confinements increases until at least the 30–34 age-group, after which illegitimate confinements are too few for the results to be convincing. The frequency of multiple births increases more rapidly with age for illegitimate than for legitimate confinements. Since confinements are in the illegitimate cases mainly among women under 20, when the chances of a multiple birth occurring are small, the total illegitimate figure for the ratio of multiple to total confinements is much smaller than the corresponding legitimate figure. The vastly different weighting of the various age-groups as between legitimate and illegitimate confinements (and that alone) is responsible for the higher showing of the ratio of multiple to total confinements in the case of legitimates than in the case of illegitimates. It is interesting to notice that these results are in striking rough conformity with the results of similar investigations in Australia. Knibbs considered multiple confinements in relation to the ages of mothers only, and found that the ratio thereof to total confinements was at a maximum at from 36 to 38 for legitimate and 35 for illegitimate confinements. He also found that the relative frequency of multiple births increased more rapidly with age among illegitimates than among legitimates.[1]

A person acquainted with the theory of probability but unacquainted with medical science and statistics might expect a priori that among four cases of twins we should find on the average one case where both were males, one where both were females, and two where the children were of opposite sexes. Twins are however of two kinds: (a) uniovular—produced by the division of a single ovum: these cases are recognized by the fact that the twins have a common chorion; (b) diovular—produced from two ova. These cases are recognized by the chorion being divided. Uniovular twins are always of the same sex; diovular twins may or may not be.

Data as to the sexes of twins are available in New Zealand as shown in Table XI.

TABLE XI

Year	Both males	Both females	Opposite sexes	Total cases
1912	114	106	109	329
1913	108	86	120	314
1914	105	116	129	350
1915	93	88	121	302
1916	99	98	126	323
1917	90	105	117	312
1918	87	95	112	294
1919	81	79	110	270
1920	128	105	132	365
1921	125	90	132	347

[1] *Mathematical Theory of Population*, pages 304 ff.

This is at the rate of 103.4 males per 100 females, a much lower rate of masculinity than prevails among births in general.

No statistics are available in New Zealand as to whether twins are produced uniovularly or diovularly in any particular case, but from the available statistics the proportion of uniovular cases can be roughly deduced by the formula devised by Knibbs, in his *Mathematical Theory of Population*, page 307. Where M = the number of pairs of males, F = the number of pairs of females, and P = the number of cases where the twins are of opposite sex, the required proportion is given by the expression $\frac{M+F-P}{M+F+P}$. For 1912–22 the New Zealand result is. 246, or .002 more than the *observed* figure for the German Empire for the years 1906–1911.

The number of cases of triplets and quadruplets, etc.—usually one or two annually—is too small to permit of detailed consideration.

STILLBIRTHS

Compulsory registration of births of stillborn children has been in force in New Zealand since March, 1913, a "stillbirth" being defined as the birth of "a child which has issued from its mother after the expiration of the twenty-eighth week of pregnancy and which was not alive at the time of such issue."

The number of stillbirths registered from 1914 onwards is given in Table XII.

TABLE XII

Year	Males	Females	Sex not stated	Totals
1914	421	244	14	679
1915	382	340	6	728
1916	383	295	4	682
1917	375	305	14	694
1918	396	300	5	701
1919	377	288	15	680
1920	467	363	10	840
1921	493	402	8	903

Stillbirths are much more frequent in multiple than in single confinements. A very large proportion of stillborn children are males, thus exemplifying the greater frailty of male foeti and infants. Both these facts are illustrated by Table XIII.

It is also of interest to inquire to what extent the ages of parents influence the proportion of stillbirths to total births. This is investigated on lines similar to those adopted in connection with the inquiry into the influence of the ages of parents on the relative frequency

TABLE XIII

Year	Females per 100 males among		Ratio of stillborn children in Confinements resulting in birth	
	Still children born	Live children born	Of a single child live or dead to total such confinements	Of more than 1 child (irrespective of whether live or dead) to total such confinements
1914	58	95	..	..
1915	89	93	..	..
1916	77	94	..	..
1917	81	97	..	..
1918	76	97	.025	.14
1919	76	95	.026	.16
1920	78	94	.026	.13
1921	82	96	.033	.10

of multiple confinements. Conjunctions in which fewer than 400 births occurred are omitted.

For nuptial confinements, it will be observed, stillbirths increase markedly with increasing age on the part of the mothers. The age of the father would appear to have little or no influence. Stillbirths are relatively much more frequent in ex-nuptial than in nuptial confinements. Especially is this true for ex-nuptial confinements where the mother is under 20. The differences between the two classes of confinements may be attributed to the fact that pregnant single women normally remain at their occupations up to the time of confinement, or nearly so, often concealing their condition from those with whom they come into contact. Married women who are pregnant receive greater care and are for the most part able to adopt a regimen of health suitable to their condition. The extreme youth of many of the mothers under 20 of illegitimate children (see above), unfitting them physiologi-

TABLE XIV

NUMBER OF STILLBIRTHS PER 100 LIVING BIRTHS, 1914-1920, CLASSIFIED ACCORDING TO THE AGES OF PARENTS

Mothers' ages in years last birthday	Fathers' ages in years last birthday									Total legitimate	Total illegitimate	Total
	Under 21	21–24	25–29	30–34	35–39	40–44	45–49	50–54	55–64			
Under 20	1.5	2.5	1.6	1.0	..	..	..	..	..	1.9	3.5	2.5
20–24	1.2	2.4	1.9	1.7	1.5	1.8	..	..	..	1.9	2.9	2.0
25–29	..	3.1	2.5	1.9	2.0	2.1	2.5	..	..	2.2	3.0	2.2
30–34	..	..	2.6	2.6	2.4	2.4	2.5	3.5	..	2.5	3.4	2.5
35–39	..	..	3.5	3.3	3.4	3.1	3.7	3.5	2.9	3.3	4.8	3.4
40–44	..	..	..	..	4.0	4.6	4.0	4.0	3.9	4.3	..	4.2
45 and over	..	..	..	..	..	..	6.1	..	..	5.3	..	5.2
Total	1.4	2.6	2.3	2.2	2.6	3.6	3.5	3.4	3.2	2.6	3.2	2.6

cally as it does for childbearing, is probably responsible for the abnormally high proportion of stillbirths among ex-nuptial confinements of girls (they can scarcely be called women) at those ages. It will now be clear that the low masculinity of illegitimate live births is wholly fictitious, being the outcome of (a) the high proportion of stillbirths among ex-nuptial confinements; and (b) the high masculinity of stillbirths.

PRINCIPLES UNDERLYING THE ISOLATION OF CYCLES AND TRENDS

By Willford I. King

The first article of this series was devoted to developing a feasible and effective method of eliminating normal seasonal variations. After these have been disposed of, the residual items are likely still to represent not a series of quantities, the movements of which are governed by some simple law, but rather a complex waiting to be analyzed.

The fluctuations found in variable numbers are commonly resultants of a vast number of forces acting simultaneously. Fortunately for the statistician, a great majority of these forces are minor in character and independent of each other. Hence, they tend to cancel each other and the net effect of all of them taken together is likely to approach zero. The observed oscillations in the variable are, as a rule, due to the action of only one or two or three major forces. Sir Isaac Newton long ago laid down the principle that when a number of physical forces act simultaneously upon a given body, each produces its own effect independent of any of the others. There seems every reason to believe that this law is equally applicable to economic phenomena in general. Fluctuations in the volume of railroad traffic may be utilized to illustrate the way in which this principle actually works. The volume of goods moving on the railroad responds in different degrees to each of the following forces:

1. Weather conditions—producing a seasonal cycle having a twelve months wave length.
2. The short business cycle of unknown origin having a wave length of some forty months.
3. Longer business cycles not yet definitely isolated. (There is some evidence that two of them have wave lengths of about seven and twenty years respectively).
4. Trends (or perhaps cycles of very long wave lengths) resulting from such forces as growth of population, geographical movements of population, or changes in habits of consumption.

Any particular type of fluctuation in which we happen to be interested can be successfully studied only when most of the other kinds of fluctuations have been eliminated. During the last year or two, much stress has been laid on the particular problem of isolating those movements which result from the phenomenon known as the business cycle. To accomplish such a purpose, it is usually necessary first to get rid of seasonal fluctuations, the best way of accomplishing which

was discussed in the last issue of this JOURNAL. But, in many instances, to do this is not enough, for the variable under consideration is affected not only by the seasonal factor but its course is also determined by two or more cyclical influences, each of which has a different wave length. In such cases, in order to isolate the effect of any particular force, it is necessary to eliminate the effects not only of seasonal influences but also of those forces giving rise to trends or cycles of other wave lengths than that of the cycle which it is desired to study.

In some ways, it is easier to eliminate seasonal fluctuations than those of any other type. This is true because they all have a known wave length of twelve months, and there is some uniformity in the changes of the weather from year to year. As brought out in the earlier paper, there appears to be no feasible method of getting rid of the actual seasonal for any particular year. The best that can be done is to dispose of the normal seasonal. The fact should be kept in mind that, when this is accomplished, all unusual or abnormal seasonal effects are still present and undiminished in magnitude.

To isolate the cyclical fluctuations, it is necessary, as stated above, not only to get rid of the seasonal movements but also to eliminate the so called trends. In most instances, there is no way of knowing in advance whether trends are segments of straight lines, of curves representing waves of great length, or of mathematical curves of some other type. These trends may be the resultants either of one or of a number of forces, some of which may be known. When we are aware that a given measurable force is present and has an important influence, the wisest procedure is to eliminate its effects first of all. For example, we may divide the production of wheat by the number of persons in the United States, thus reducing the figures to a per capita basis and causing the marked upward slope to disappear from the curve. Similarly, we may cause the trend of business failures more closely to approach the horizontal by dividing the number of failures by the number of firms from which reports are secured. Very commonly, however, even after all known and measurable forces of this type have been eliminated, the variable will still have a tendency either to oscillate or gradually to rise or fall. Nevertheless, the problem is simplified by disposing of the known components.

When all measurable long-time factors have been eliminated and the variable still does not proceed along a horizontal line but continues to tend upward or downward, what is the first step to be taken in eliminating the residual trend? We have the option of using mathematical or non-mathematical devices. Which shall be applied? The answer is that each method is best in its proper place.

A caution which it would at first seem unnecessary to give but which experience shows is really most essential is that mathematical processes can never be substituted for knowledge of the facts or for the ordinary processes of logic. In other words, mathematics must never be divorced from but rather must always be coupled with every bit of information available. Unfortunately, examples are all too abundant in which this rule has been entirely ignored. For instance, in those cases in which a trend has been broken by some unusual force, it is absurd to try to fit either a single smooth curve or a straight line to the two or more remaining fragments. Such a break in the trend is typified by that which occurred in the wholesale price index between the years 1914 and 1920. For some time before 1914, the trend of the price level had been primarily dependent upon the fall in the cost of producing gold. Then came the enormous paper inflation which elevated the entire level to a new plane. Manifestly, the trend which correctly portrayed changes in the cost of producing gold had no connection with the new trend which depended upon the rapidity with which the various governments could print paper money.

It is then sheer folly to search for a smooth line which will show *the* trend of prices from 1900 to 1924, for no such continuous trend exists. It is equally absurd to assume a continuous trend for figures representing production measured in terms of value when, as one would naturally expect, the price cataclysm has diverted the curve to an entirely new course.

In either of these cases, the obviously correct method is to fit a separate trend to each segment. Yet instances are numerous in which statisticians have attempted by means of one sweeping line to include the unrelated trends of the two periods.

The fact should always be kept in mind that the use of a mathematical curve implies a belief that the force or forces dominating the variable act according to mathematical law. Only in such instances is the use of a mathematical curve appropriate, for its chief advantage is that it is easy to project it into the future, since its course is definitely predetermined. Nothing is more absurd than to assume that a mathematical curve represents the actual trend when inspection shows that the curve does not fit the data satisfactorily. Just so long as a freehand curve can be drawn that manifestly is a better fit than a mathematical curve, it is certain that the latter cannot legitimately be used as synonymous with the trend. Once the formula correctly representing the law governing the dominant force or forces is discovered, the fact will be revealed, for it will be apparent to the eye that this correct mathematical curve fits the data better than any freehand curve that is tried as a substitute.

This statement may be challenged by the more mathematically inclined, for it is manifestly equivalent to saying that the eye is the ultimate court of appeal as to the correctness of the location of the trend. The fact remains, however, that even the most skilled mathematicans usually decide by observation whether or not a mathematical formula is giving satisfactory results. In practice, they, as well as the rest of us, are inclined to believe that judgment is after all superior to any rule of thumb procedure.

There has been considerable discussion in the past concerning the relative merits of curves and straight lines as representations of trends. Advocates of the use of the straight line point out that it has the marked advantage of being easy to project into the future. That this is an important advantage cannot be denied. Sometimes, it is feasible by the use of a simple mathematical process to convert the raw data, which when plotted, lie along a curve, into a form which will reduce the graph to a straight line. During the last few years, we have heard much of the use of the ratio chart. While, perhaps, many persons who have used logarithmic curves may not have analyzed clearly the advantages found therein, the fact remains that the most important end served by the use of logarithms is to straighten out curves representing historical variables which tend to increase each year by the same fraction. In a country like the United States, where the population has been growing rapidly, many curves of this type exist, because the movements of thousands of other things are indirectly dependent upon population growth. Population tends to grow during each interval of time by proportional accretions. When a variable is of this type, it is obvious that when it is plotted logarithmically, it will yield a straight line. For such phenomena, therefore, ratio charts manifestly are admirably suited, since they tend to make the trends approximate straight lines. However, this is far from saying that, for other types of data, ratio charts are better than those plotted on a natural scale. If the data are not of the particular type just mentioned, the logarithmic treatment is just as likely to make matters worse as better. If the curve is convex on the upper side, the use of logarithms will manifestly increase the difficulties of extrapolation. The fact which should be emphasized is that, in the determination of trends, logarithms have no inherent virtue except for the special purpose described above. If, in any other type of curve, it is found that squaring or cubing the figures, extracting their square roots or cube roots, or subjecting them to any other similar mathematical process, would tend to throw the data into a straight line, then the process that accomplishes this result should be adopted just as readily as is

the logarithmic treatment when it serves the same end. The supreme test of the merit of any such device is: "Does it work?"

When no means can be discovered of reducing data to a form in which the straight line is the best fit, then it is absurd to use a straight line as a trend. If a mathematical curve that fits the data cannot be discovered, it is infinitely more logical and more scientific to use a "French curve" as an aid in drawing, or even to resort to a purely free-hand curve for interpolation or extrapolation than to stick to the straight line trend or to some other mathematical curve merely because it is possible to use some such mathematical process as the method of least squares in determining the course of the curve chosen. The use of mathematical curves that do not fit can only be ascribed to an attempt to delude one's self into a belief in the accuracy of the results or else to an effort "to pull the wool over the eyes" of others and to convince them that the computer is a real scientist. The truth is, of course, that by such a procedure, the reverse is clearly demonstrated.

In the eyes of many statisticians, the method of least squares is venerated in a way which leads one to suspect that it is believed to be either sacred or coupled up in some way with "natural law." Such persons feel certain that when they have gone through the mechanics of obtaining a line or a curve by this process, they have thereby impregnably entrenched themselves, that they have found the "Ultima Thule," the exact location where the correct line must lie.

As a matter of fact, the least square method is a reasonably simple way of locating a straight line which, in most cases, seems to approximate the trend as closely as a straight line can be expected to do. In some instances, however, it appears to the eye that the line thus located has a wrong slope and one is thereby led to inquire whether or not

Ordinate	Abscissa	xy	x^2
y	x		
0	−6	0	36
0	−5	0	25
0	−4	0	16
0	−3	0	9
0	−2	0	4
0	−1	0	1
0	0	0	0
−1	+1	−1	1
0	+2	0	4
0	+3	0	9
+1	+4	+4	16
0	+5	0	25
0	+6	0	36
		3	182

there is a sound mathematical defense for the method. A very limited amount of experimenting is sufficient to show that the procedure is technically imperfect. The example on the opposite page will suffice for the purpose.

From the above computation it appears that the slope of the least square straight line is at an angle whose tangent is 3/182. The angle is about 1 degree.

If, when plotted, eleven out of thirteen recorded items in a historical variable lie along a horizontal line and if one of the others is one unit above the line and the other one unit below, the most logical conclusion to draw is that the trend is a horizontal line and that the two stray items have been moved from their normal positions not at all by the dominant force but wholly by other forces. Since it is the effect of the dominant force which one uses the trend to portray, the effects of other forces ought to be given no weight whatever. The method of least squares, however, gives weight to these stray items and gives most weight to the one furthest from the point of origin. As a result, in the above example, we arrive at a false trend sloping upward at an angle having a tangent of 3/182 and equalling about 1 degree. True, this error is very small, but it shows that the method is not a precise one. When the item having the large deviation happens to lie at a considerable distance from the point of origin, the error may become large enough to be of importance.

The ideal trend is the one that best portrays the effect of the dominant force. Is there a better method than that of least squares for locating this line? Were there an infinite number of scattered points, the course of the true trend would be through the modes of those points lying on common ordinates. Since, in practice, there is only one point on an ordinate, this ideal can, at best, only be approached by thinking in terms of a moving mode. As a matter of fact, it is wholly impracticable to locate a moving mode. What is the next best thing? The answer seems to be: "Use a moving median." We know that the median normally diverges from the mode by only about two-thirds as much as the arithmetic average; hence a line passing through the medians tends to approach one striking the modes.

If one seeks only a straight line, it is sufficient to divide the historigram in the middle by a vertical line and then draw the trend so that both on the left and the right sides of this line the items above and below the trend are equal in numbers. This is approximately the method used by Sir Francis Galton for locating regression lines.

The cases in which a straight line suffices to portray the trend appear to be in a decided minority. As a rule, then, it is best to locate a

series of points by aid of a moving median and then to draw a curve through the vicinity of these points. The moving median has the advantage over the moving average that it is unaffected by widely-deviating items. In certain instances, however, its use does not produce satisfactory results. For example, when the class intervals are not wide enough to include at least seven items each, and the fluctuations in the items are large, the course of the moving median may be so erratic that it does not determine the trend with any reasonable degree of precision. Furthermore, if the moving median is used with unadjusted monthly data having a wide seasonal variation in one direction for two or three months and a small seasonal swing in the opposite direction for the other nine or ten months, the line determined is influenced too much by the smaller deviations and, as a result, it will fail to pass through the annual totals. This difficulty can be avoided if the seasonal fluctuations are eliminated before applying the moving median.

Both the moving average and the moving median share the weakness of failing to reach the terminal date, and this is a serious matter when trends are to be continued into the future. Another defect common to both is that when the trend changes its course in an abrupt fashion the true sharpness of the angle is not revealed, but, on the contrary, the figures indicate a curve of relatively large radius.

Whether the moving average or the moving median is used, it is almost always necessary to smooth the crude curve thus obtained in order to arrive at a line which approaches the true trend. To neglect this last step is to fail to make the most of the data. Such smoothing may be done by mathematical or non-mathematical means—the choice being largely a matter of taste.

There are cases in which, as a means of locating the true trend, the simple method just outlined is distinctly inferior to more intricate mathematical devices. These cases are mainly those in which records are abundant and the same force has been dominant for a considerable period of time so that the law which it obeys has had time to manifest itself. Since such cases are none too common, the non-mathematical statistician rarely is called upon to lie awake nights fearing that some mathematical expert will demolish by his perfected methods the rough and ready trends located by aid of moving medians and "French curves". He is wiser if he ceases to worry and decides instead to rejoice with his mathematical rival in those occasional instances in which the latter proves the superiority of his much more complicated and time consuming methods.

SUMMARY

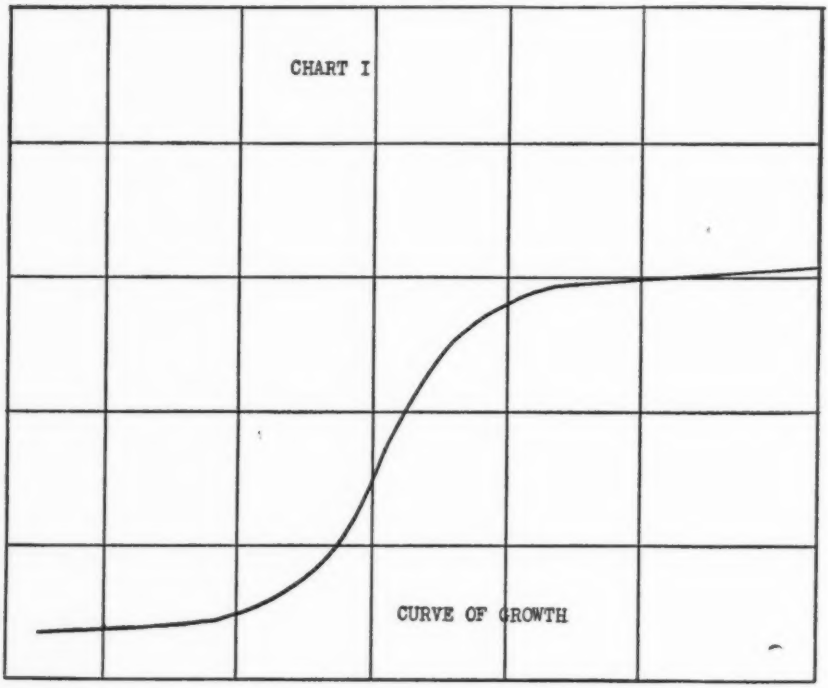
1. As a rule, historical variables are the resultants of numerous forces acting simultaneously.
2. In most instances, the course of the variable is dominated by a very small number of these forces.
3. Long time trends are the resultants of those forces which neither act irregularly nor have short wave lengths.
4. The line called "the trend" represents an attempt to eliminate variations due to fortuitous occurrences or to forces having short wave lengths. The true course of the trend may be conceived of as lying along the course traversed by a moving mode.
5. If a trend chances to be a straight line, it can usually be located with greater accuracy by the median method than by the method of least squares.
6. It is often advantageous to rid the curve of the effects of known forces—as for example by reducing the data to a per capita basis.
7. Straight lines are easiest to produce into the future—hence it is frequently desirable to use logarithms or other mathematical devices in order artificially to force the data to lie approximately along a straight line.
8. The moving median is the most generally useful method yet devised for locating the trend. The rough original results should always be smoothed.
9. When the fluctuations in a variable are not too irregular and when records are available covering a long period of time, the most refined mathematical methods give the most accurate results.
10. The eye is the court of last resort in determining whether or not the true course of the trend has been well or badly determined.

GROWTH CURVES AND RAILWAY TRAFFIC

By Leroy E. Peabody

If the various types of growth curves may be applied to railway traffic, it should be possible to evaluate the constants and find the rate of growth of traffic at various points (or dates) along the curve. Most of the growth curves have points of maximum rate, or inflection points; and a knowledge of the location of these points of maximum rate of growth is of value.

It is natural to expect that traffic growth has been more rapid in those parts of the country that were settled and developed early. The point of maximum rate of growth may also be expected earlier in the older sections. Two types[1] of curves have been used in the present paper in an attempt to locate approximately the date of maximum rate of growth of the ton-mileage of the three districts of the United States.



[1] Three other types of growth curves were considered and found unsuitable for this purpose, having maximum points within the range of the data and no inflection points.

It is interesting to know whether or not the rate of growth of ton-mileage for the Eastern District has reached its maximum and to know the date of that maximum. Have all districts reached the maximum point, or may some be expected to continue with increasing rates of growth? It is conceivable that the motor-truck may some day completely monopolize freight traffic; in which case, the rates of growth of ton-mileage will become zero. Such an assumption seems unlikely to say the least.

Tons of revenue freight carried one mile have been separated into districts in the reports of the Interstate Commerce Commission within a few years only. The earlier reports of the commission divided the country into ten groups. Of these, groups I, II and III correspond closely to the present Eastern District, which includes roughly that part of the country east of the Mississippi River and north of the Potomac. Groups IV and V are the same as the present Southern District, which is the section south of the Potomac and East of the Mississippi. Groups VI to X correspond to the present Western District and include the remainder of the country. The ton-mileage by districts since 1890 is presented in Table I.

TYPES OF CURVES USED

The types of curves used are of the same general nature. Briefly, the general assumptions with regard to the data treated are: A changing rate of growth; at first slow, but gradually increasing to a point

TABLE I

Year	Ton Miles (billions)			Year	Ton Miles (billions)		
	Southern District	Western District	Eastern District		Southern District	Western District	Eastern District
1890	7.7	26	43	1906	27.3	881	107
91	8.4	28	44	07	28.8	90	118
92	8.4	30	50	08	27.0	83	108
93	9.7	33	51	09	28.6	84	106
94	9.3	28	43				
1895	10.1	27	48	1910	33.8	96	125
96	11.1	31	53	11	41.7	81	131
97	11.7	33	51	12	44.3	85	135
98	13.7	40	60	13	48.5	99	154
99	14.6	42	67	14	50.1	94	144
1900	17.1	49	75	1915	47.8	92	137
01	17.9	54	76	16	62.3	125	179
02	19.4	59	79	17	68.4	138	188
03	20.9	65	87	18	72.1	142	191
04	21.7	66	87	19	64.0	130	170
1905	23.6	69	94	1920	76.9	145	189
				21	59.7	109	139
				22	69.6	119	151

where the rate becomes a maximum. After this point the rate becomes steadily slower, until finally the curve becomes nearly horizontal. The underlying assumption permitting the use of these curves is that the development shown by the data fitted is typical of the whole movement. Aside from purely temporary rates for a few years only, it assumes that after a point of maximum rate of growth is reached, the rate of growth will continually decrease. The general form of the curve is sketched in Chart I.

The Gompertz curve is the first of this type. It's equation is

$$y = ab^{c^x} \text{ or } \log y = \log a + c^x \log b.$$

The rate of growth is given by

$$\frac{1}{y}\frac{dy}{dx} = c^x b', \; b' = \log_e b \cdot \log_e c.$$

The point of maximum rate of growth is determined from the equation

$$c^x = \frac{-1}{\log_e b}.$$

This curve will have no asymptote if c is greater than unity.

The equation of the Makeham curve

$$y = k\, s^x\, g^{c^x}$$

and its derivative

$$\frac{1}{y}\frac{dy}{dx} = s' + c^x\, g', \; s' = \log_e s, \; g' = \log_e g \cdot \log_e c.$$

show that is of the same nature as the Gompertz curve but that it will have no asymptote. It is somewhat more flexible that the Gompertz curve, due to the extra constant, and might be expected to give a better fit to the data. The point of maximum rate of growth is obtained by solution of the equation

$$c^x = \frac{-(c' + 2s') \pm \sqrt{c'^2 + 4c'\, s'}}{2\, c'\, g'}$$

where s', c' and g' are the logarithms to the base e of s, c, and g. It will be seen that this curve has no point of maximum rate of growth if $4c's'$ is negative and greater than c'^2.

METHODS OF FITTING CURVES

The methods used in fitting these curves have been the summation process described by Raymond B. Prescott in the Quarterly Publication of the American Statistical Association December 1922, and the

method of moments. The constants for the Gompertz curve may be evaluated by the method of moments as follows.[1]

$$\log y = \log a + c^x \log b$$

or

$$Y = A + B^{2nx/l}$$

where $1 = \text{range}$, with the origin at the middle of the range, $e^{2n} = c$, $A = \log a$, $B = \log b$. The constants may be found from the following equations.

$$n = \frac{12D\mu_2/l^3 - D/l}{8D\mu_3/l - 2D\mu_1/l} \tag{1}$$

where $D = \Sigma y$, $D\mu_2 = \Sigma x^2 y$, $D\mu_3 = \Sigma x^3 y$, $D\mu_1 = \Sigma xy$.

$$e^{2n} = c. \tag{2}$$

$$\frac{2D\mu_1}{l} = B\left(\frac{\cosh n}{n} - \frac{\sinh n}{n^2}\right). \tag{3}$$

$$\frac{D}{l} = A + B\frac{\sinh n}{n}. \tag{4}$$

The equations just given were obtained by setting the moments of the curve equal to the moments of the data.

For the Makeham curve the constants are obtained as follows.

$$y = k\, s^x\, g^{c^x}$$

$$y = k'\, s'^{x/l}\, (g')^{c'^{x/l}}$$

$$Y = K + S^{x/l} + G\, e^{2nx/l}$$

where $K = \log k'$, $S = \log s'$, $G = \log g'$, $e^{2n} = c'$.

$$n = \frac{2\beta_1}{1-\beta_2}, \text{ where } \beta_1 = \frac{a_3 - a_1}{a_2 - a_0},\ \beta^2 = \frac{a_4 - a_2}{a_2 - a_0} \tag{1}$$

$$a_0 = \frac{\Sigma y}{l},$$

$$a_1 = \frac{12}{l^2}\Sigma xy,\ a_2 = \frac{12}{l^3}\Sigma x^2 y,\ a_3 = \frac{80}{l^4}\Sigma x^3 y,$$

G may be found from the equation

$$a_2 - a_0 = g\left(\frac{2\sinh n}{n} - \frac{6\cosh n}{n^2} + \frac{6\sinh n}{n^3}\right). \tag{2}$$

K from

$$a_0 = K + G\frac{\sinh n}{n}. \tag{3}$$

S from

$$a_1 = S + 6G\left(\frac{\cosh n}{n} - \frac{\sinh n}{n^2}\right) \tag{4}$$

[1] I am indebted to Mr. H. Schuls of the National Bureau of Economic Research for the method here given.

To compare the curves it was decided to apply them first to the miles of main track in operation in the United States from 1830 to 1921. This series is the longest that is available. The results are stated briefly for each curve as follows.

Gompertz Curve	*Makeham Curve*
	$\log s = -0.00349$
$c = 0.9692$	$\log c = -0.0128$
$\log b = -3.1784$	$\log g = -3.6115$
$\log a = 5.6384$	$\log k = 6.0030$

When the curves are fitted to the data the Makeham curve gives the best result. The standard deviations of the actual values from the graduated values were for the

Makeham curve	8199
Gompertz curve	8817

The mean mileage is 102,774.

The points of maximum growth were 1884 for the Gompertz curve and 1888 for the Makeham curve. For the purpose of finding the point of maximum growth there seems little choice between the curves considered. The building in the decade ending in 1840 and for subsequent decades is shown below:

1840	2,265 miles
1850	5,046 miles
1860	20,110 miles
1870	16,090 miles
1880	44,291 miles
1890	75,796 miles
1900	29,749 miles
1910	46,947 miles
1920	12,522 miles

so that the years from 1880 to 1890 are those of greatest growth of railway mileage.

Table I shows that the Eastern District maintains the greatest volume of traffic continuously throughout the period 1890 to 1922, with the Western Districts and Southern Districts following in order. The figures of Table I have been plotted upon a logarithmic chart to compare the rates of increase in the several districts. To the graphs have been fitted least-square trend lines. If the traffic in each district were increasing at the same rate, these lines of trend would be parallel. The slopes of the trend lines for the three districts were found to be as follows:

Eastern District	0.02147
Western District	0.02417
Southern District	0.03321

Thus the order of the districts ranked by their rates of increase is: Southern, Western and Eastern.

The Gompertz curve applied to the Southern District using the summation method, gave the following equation:

$$\log y = 11.2445 - 10.3854\, c^x, \qquad c = 0.9966,$$

where y is ton-miles in billions, x the number of years after 1890. The standard deviation from this curve was 5.36 billions, mean 105.45 billions. If the abnormally low years 1921, 1922 are excepted, the standard deviation is 3.21 billions. This curve has a point of maximum rate of growth at $x = 931$. If the method of moments be used, the standard deviation is 5.88 billions. The constants are $n = 0.17483$, $c = 1.4186$, $\log b = 2.88002$, $\log a = -1.48809$. This curve fitted by the method of moments, has no asymptote nor point of maximum rate of growth and is, therefore, unsuitable for the present purpose.

The Makeham curve applied to the Southern District using the summation method, gives $\log s = 0.0296$, $\log c = -0.02243$, $\log g = -0.1764$, $\log k = 1.0207$. The standard deviation of the actual values from the graduated values is 5.43, as compared with 5.36 for the Gompertz curve. This curve has no point of maximum rate of growth. When the method of moments is used, meaningless results are obtained.

The Gompertz curve applied to the Eastern District using the summation method gave the equation.

$$\log y = 2.7477 - 1.1838 c^x, \qquad c = 0.9714.$$

The standard deviation was 14.2 billions, mean 105.45 billions. This curve has a point of maximum growth at $x = 35$, origin at 1890. The method of moments gave a curve with no point of maximum rate of growth.

The Makeham curve applied to the Eastern District using the summation method gave the following values for the constants: $c = 1.248$, $\log s = 0.02639$, $\log g = -0.000189$, $\log k = 1.58586$. The standard deviation of this curve is 10.39 billions, as compared with 14.2 billions for the Gompertz curve. It is, therefore, a better fit for the ton-mileage of the Eastern District from 1890 to 1922. This curve has the disadvantage of a maximum point (not a point of maximum rate of growth) at $x = 29$, or at the year 1919.

It is seen that the Gompertz curve fitted by the summation method gives the most satisfactory results. This curve has a point of maximum rate of growth for all three districts and fits the data more closely than if determined by the method of moments.

The dates for the points of maximum rate of growth for the 3 districts are given below:

Eastern District	1925
Western District	1909
Southern District	2821

as obtained from the equations of the curves fitted to the data.

There are two surprises: that the Western District is first in passing its point of maximum rate of growth, and that the Southern District will not reach this point for a long period of years. The dates for the Southern District above given may not be strictly interpreted as there has been, since 1890, so little change in the rate of growth of ton-mileage in the Southern District. The logarithmic chart of the Southern District is nearly a straight line. It seems likely that slight changes in the rate of growth in the Southern District would materially cut down the length of time to the maximum. There is as yet no indication of a "tailing off" of the curve—no indication of a decreasing rate. In the case of the Western District there is a very noticeable slowing up in the rate of growth beginning about 1908. A correction of the ton-mileage with an allowance for traffic diverted from the railroads to the Panama Canal, has but little effect upon the point of maximum rate for the Western District. In fact, the date given above includes such allowance. If no allowance were made, the date for the point of maximum rate in the Western District is 1905. In the Eastern District there is also a slowing up of the rate of growth about 1908, but to a smaller degree than in the West.

If the correlation coefficients for the deviations from the Gompertz curves, fitted by the summation method, are computed we get

$$r = 0.73 \pm 0.055 \text{ between the East and the West;}$$

$$r = 0.54 \pm 0.083 \text{ between the South and the West.}$$

Data for the years prior to 1890, say back to 1870, or even to 1880, would be very helpful. The longer series would minimize the effects of the tremendous increases in every district after 1890, upon the constants in the equations.

TABLE II

Year	Actual total	Calculated total	Error	Percentage error
1890	76	63	−13	−17
1900	142	141	− 1	− 1
1910	255	247	− 8	− 3
1920	410*	377*	−33	− 8

* Class I Roads.

Another test of the accuracy of the curves is secured by comparing the total ton-mileage of the country with the total obtained by adding the values calculated from the equations for each district.

The traffic of the year 1920 was abnormally high; about equal to the 1923 traffic.

Still another check upon the curves used in each district may be obtained by computing the ton-mileage for future years from the equations, again subject to the reservations previously made. For 1933 the ton-mileage will be:

Eastern District.....	256 millions
Western District.....	155 millions
Southern District.....	189 millions
Total.....	600 millions

This estimate for the 1933 traffic is 40 billions more than the estimate of the Chamber of Commerce of the United States and 117 billions less than the estimate of Mr. John Balch Blood published in the *Railway Age* of February 9, 1924. The fact that the Southern District estimate for 1933 is greater than that of the Western District is significant, and it agrees with what has been said previously—that Southern District traffic is increasing at a faster rate than the Western District.

CONCLUSIONS

Apparently, the simpler Gompertz curve gives more satisfactory results than the more flexible Makeham curve, the latter having no point of inflection in several instances. In one case (that of the Eastern District), the Makeham curve had the disadvantage of a maximum point within the range of the data, and in all cases the standard deviations of the residuals were greater from the Makeham curve than from the Gompertz curve.

The less involved method of fitting by summation gives in each case more satisfactory results than the method of moments. This is due, in part perhaps, to the errors introduced by taking moments of third and fourth orders.

COST OF LIVING STATISTICS OF U. S. BUREAU OF LABOR STATISTICS AND THE NATIONAL INDUSTRIAL CONFERENCE BOARD

COMPARISON OF METHODS USED IN THE COLLECTION AND COMPUTATION OF COST OF LIVING FIGURES

By Elma B. Carr

The United States Bureau of Labor Statistics in a recent investigation to determine to what extent cost of living figures have been used in the adjustment of wages found that the two series of index numbers most generally used were those of that Bureau and of the National Industrial Conference Board.[1]

It is interesting to compare the methods of these organizations in the collection and compilation of cost of living figures published by them. These comparisons have been made in this article under the following headings: I, Sources of material; II, Articles on which prices are secured; III, Weightings used for individual items; IV, Weightings used for the different groups.

I. SOURCES OF MATERIAL

United States Bureau of Labor Statistics	National Industrial Conference Board
For the United States	For the United States[2]
Average prices are computed from retail prices, obtained by agents and questionnaire, in 32 cities[3] for coal, gas, electricity, wood, clothing, furniture, and furnishings, and miscellaneous articles, and in 51 cities for food.[4]	

[1] U. S. Department of Labor. Bureau of Labor Statistics. "Use of Cost of Living Figures in Wage Adjustments," Bul. No. 369 (now in press).

[2] The National Industrial Conference Board does not publish prices for individual cities. The prices are obtained from a varying number of cities from time to time, and are published as representative of the country as a whole.

[3] Atlanta, Baltimore, Birmingham, Boston, Buffalo, Chicago, Cincinnati, Cleveland, Denver, Detroit, Houston, Indianapolis, Jacksonville, Kansas City, Los Angeles, Memphis, Minneapolis, Mobile, New Orleans, New York, Norfolk, Philadelphia, Pittsburgh, Portland, Me., Portland, Ore., Richmond, Va., St. Louis, San Francisco, Savannah, Scranton, Seattle, and Washington.

[4] Food prices are obtained by the U. S. Bureau of Labor Statistics monthly from 51 cities. Although the change in the cost of food in each of the 32 cities is computed from changes in retail food prices in that city, changes in the cost of food for the United States are based on changes in retail food prices in 51 cities. Prices are quoted for each of these cities monthly in the *Monthly Labor Review*.

United States Bureau of Labor Statistics—Cont.

For each of 32 cities [3]

a. Food—prices secured from retail grocers, butchers, bakers, dairy men by questionnaire.[5]

b. Clothing—prices secured from retail clothing dealers by agents of Bureau.

c. Rent—prices secured from books of real estate dealers and owners of dwellings by agents of Bureau.

d. Fuel and light—prices secured by questionnaires sent to retail coal dealers, gas and electric light companies, wood dealers.[5]

e. Furniture and furnishings—prices secured from retail dealers handling these articles, by agents of Bureau.

f. Sundries—prices secured by agents of Bureau on:
- Street car fare from street car companies.
- Movies from moving picture theatres.
- Newspapers from publishers.
- Doctors' fees from doctors.
- Medicine from druggists.
- Hospital care from hospitals.
- Dental charges from dentists.
- Spectacles from opticians.
- Laundry from laundries.
- Cleaning supplies and kerosene from grocers.
- Barber service from barbers.
- Toilet articles and preparations from druggists.

National Industrial Conference Board—Cont.

a. Food—figures of United States Bureau of Labor Statistics for 51 cities.

b. Clothing—prices secured from retail clothing dealers by questionnaire.

c. Rent—estimates of prices being charged for four- or five-room house with bath, secured from real estate brokers and boards, boards of trade, chambers of commerce, trust departments of banks, bureaus of municipal research, housing associations, charitable and welfare organizations, by questionnaire.[6]

d. Fuel and light [7]—prices secured by questionnaire on:
- Coal—from retail coal dealers;
- Gas—from people furnishing information on rent.

e. Sundries:

(1) Estimates of changes in price of:
- Furniture, carpets and rugs, brooms and brushes, china and crockery, glassware, and kitchen utensils—secured by questionnaire from retail dealers supplying information on clothing and who handle these articles.
- Moving pictures—secured by correspondence to moving picture theatres and motion picture owners' associations [8] in about 70 cities.
- Candy—secured by correspondence from headquarters of large distributors of well known popular brands.

[5] Firms have previously been selected by agents of the Bureau, and brands or grades upon which retailer is requested to report have been decided upon by agent and noted in retailer's book. These firms are also visited from time to time by agents of the Bureau.

[6] Prices are often reported as a range.

[7] In November of each year, rates, extra charges and discounts for both gas and electricity are obtained from public service commissions of the separate states, or in some cases from the mayors of the cities, for each city in the United States with a population of 50,000 and over, and from several smaller places.

[8] In some cases, these associations cover several cities.

United States Bureau of Labor Statistics—Cont.

Telephone rates from telephone companies.

Tobacco from drug stores, cigar stores, and news stands.

National Industrial Conference Board—Cont.

Tobacco—secured by correspondence from headquarters of large distributors of well known popular priced goods.

Drugs—secured by correspondence from headquarters of large chain store distributors.

Reading material—secured by correspondence from newspaper agencies in New York and magazine agencies in New York and several other cities.

Medical care—secured by correspondence from a varying number of physicians in a varying number of cities.

Insurance, charitable and church contributions, gifts, and organization dues—computed from the change in the cost of other items of the cost of living.[9]

(2) Prices of:

Street car fares—secured by questionnaire from people furnishing information on rent.

Cotton yard goods—secured by questionnaire from retail clothing dealers.

II. ARTICLES ON WHICH PRICES ARE SECURED [10]

For each of 32 cities [3]

a. Clothing:

Women:

House dresses, kimono, cotton envelope chemise, cotton nightgown, corset covers or camisoles, brassières, corsets, cotton stockings, silk stockings, silk waist, silk for dress, voile, organdie, gingham for dresses, gingham for apron, wool suit, wool coat, wool dress, winter petticoat or princess slip, summer petticoat or princess slip, winter union suit, sum-

For a varying number of cities: [2]

a. Clothing:

Women:

Hosiery, knit vest, muslin combinations, suit, coat, cotton blouses, shoes, cape kid gloves, velvet hat, straw hat.

[9] The National Industrial Conference Board states that the allowance for these items has been increased by approximately 10 per cent more than shown in the increase in the other items of the cost of living and has been decreased by approximately 10 per cent less than the decrease shown in the other items of the cost of living. It is impossible to check this statement from the figures.

[10] The food items are not listed in this article. The United States Bureau of Labor Statistics uses changes in the prices of 43 food articles in each of the 32 cities referred to for the purpose of obtaining the variation in the cost of food in the respective cities; the figures for the variation in the cost of food for the United States as a whole are based upon changes in prices of the same food articles in 51 cities. (*Monthly Labor Review*, March, 1921, p. 26.)

United States Bureau of Labor Statistics—Cont.	National Industrial Conference Board—Cont.
mer union suit, rubbers, low shoes, sewed half soles and heels, rubber heels. Six-year-old girl: Wool coat, serge for dress, gingham for dresses, gingham for apron, winter petticoat or bloomers, summer petticoat or princess slip, summer drawers or bloomers, summer nightgowns, winter nightgowns, summer underwaists, winter underwaists, summer vests, winter union suits, low shoes, high shoes, cotton stockings, rubbers. Men: Overcoat, winter suit, light weight suit, Palm Beach or mohair suit, overalls or work trousers, cotton shirts, felt hat, straw hat, cap, summer union suits, winter union suits, light weight union suits, pajamas or nightshirts, cotton socks, collars, neckties, low shoes, high shoes, rubbers, (sewed) whole soles and heels, (sewed) half soles and heels. Twelve-year-old boy: Overcoat or mackinaw, wool suit, cotton trousers, wool trousers, cotton shirts or blouses, summer union suits, winter union suits, pajamas or nightshirts, cap, neckties, cotton stockings, (sewed) high shoes, half soles and heels.	Men: Hosiery, knit union suit, suit, overcoat, work shirts, negligee shirts, overalls, shoes, dog skin gloves, felt hat.
b. Rents: Figures based on rentals of from 400 to 2,000 houses in each of 32 cities.	b. Rents: Figures based on average price of rents given in a varying number of replies from a varying number of cities, each month.
c. Furniture and furnishings: Wool rugs, grass rugs, linoleum, living room and dining room chairs, library or living room tables, dining room and kitchen tables, sanitary or bed couches, dressers, chiffoniers, buffets, bedsteads, bed springs, mattresses, baby carriages, pillow cases, sheets, cotton tablecloths, cotton towels, cotton blankets, wool blankets, comforts, cook stoves, heating stoves, brooms, sewing machines, and refrigerators.	c. Furniture and furnishings:[11] Estimates of changes obtained on furniture, carpets and rugs, brooms and brushes, china and crockery, glassware and kitchen utensils, but not on individual items within these groups. Changes in price of percale, gingham, longcloth, fruit of the loom, and voile used to represent change in price of house furnishings.

[11] Changes in the price of furniture and furnishings are included with sundries.

United States Bureau of Labor Statistics—Cont.

d. Fuel and light:
Anthracite and bituminous coal, wood, gas, kerosene, and electricity.

e. Miscellaneous items:
Street car fares; movies; newspapers, daily and Sunday; doctor's fees—office visit, house visit, obstretrical case; medicine—calomel tablets, aspirin tablets, castor oil, quinine pills, standard prescriptions, liquid and capsule or pill; hospital, pay ward; dentist—filling, crown, plate; spectacles and separate lens; laundry—men's stiff collars and shirts, sheets, turkish towels, flat work; cleaning supplies—soap, small and laundry, soap powder, cleaning powder; barber—shave and hair cut; toilet articles—toothbrush, toilet soap, shaving stick, tooth powder or paste, talcum powder, vaseline; telephone; tobacco—cigar, cigarettes, cigarette tobacco, pipe tobacco, plug tobacco.

National Industrial Conference Board—Cont.

d. Fuel and light:
Anthracite and bituminous coal, gas, and electricity (in November only of each year).

e. Miscellaneous items:
Prices secured for street car fares and five items of cotton yard goods. Estimates of changes in price secured for furniture, carpets and rugs, brooms and brushes, china and crockery, glassware and kitchen utensils, moving pictures, candy, ice cream, tobacco, drugs, toilet soaps, reading material, medical care (doctor's fees and drugs), insurance, charitable and church contributions and organization dues.

III. WEIGHTINGS USED FOR INDIVIDUAL ITEMS

For each of 32 cities

a. Clothing:
At least four quotations from each city are secured on each item listed below. From these quotations an average price is computed for each item in each city. This average price is multiplied by the average family consumption of each item and an annual cost obtained. A total is made of all of these aggregate costs of individual items. The percentage change in the aggregate cost of all the items of clothing in each city in the current month as compared with the aggregate cost of the same articles of clothing for the same city in the preceding month is computed. For the United States, the total aggregate cost for clothing in each of the 32 cities in the current month is

For a varying number of cities

a. Clothing:
From the prices reported by the dealers in the various cities, an average is made for each item for the country as a whole. (Averages are not computed for the individual cities.) The percentage change in the average price of each item in the current month as compared with the price in the preceding month is computed. The percentage change in each item since the preceding month is then applied to the percentage change previously computed for the preceding month compared with 1914. These new percentage changes for each item are then applied to the prices allotted to the different items for 1914 in the budgets. Thus the cost for each item in the

United States Bureau of Labor Statistics—Cont.

added, and the percentage change as compared with a similar total cost for the preceding month is computed. The figures representing the average family consumption of each item were obtained from the 1918–19 investigation, referred to above.[12] Below is shown the weighting attached to each article:

TABLE I
WOMEN'S CLOTHING

Article	Quantity Weighting
Suits, wool	½
Coats, wool	½
Dresses, wool	½
Dresses, silk, yds.	4
Dresses, voile, organdie, gingham (2), yds.	10
Waists, silk	½
House dresses	2
Kimonos	½
Apron, gingham (1), yds.	2
Petticoat or princess slip, summer	1
Petticoat or princess slip, winter	1
Envelope chemise, cotton	2
Corset covers or camisole	2
Brassières	2
Corsets	2
Nightgowns, cotton	2
Union suits, summer	2
Union suits, winter	1
Stockings, cotton	4
Stockings, silk	4
Shoes, low	2
Rubbers	1
Half soles and heels	1
Rubber heels	3

National Industrial Conference Board—Cont.

budget for the current month is obtained and totalled. This total cost of the budget in the current month is compared with the total cost of the budget for 1914, and the percentage change computed. Below are given the budgets to which the changes in prices are applied:

TABLE II
WOMEN'S CLOTHING

Article	1914 price
Coat or suit	$15.00
Woolen dress	5.00 [13]
Woolen skirt	2.00 [13]
Cotton skirts (2)	2.00 [14]
Waists (4)	4.00
House dresses (2)	2.00 [14]
Aprons (3)	.90 [14]
Shoes and repairs	6.00 [16]
Overshoes [15]	.65 [16]
Corsets	2.00 [17]
Hosiery	1.50
Summer union suits (3)	1.00
Winter union suits (3) [18]	2.25
Muslin underwear, 4 sets	3.20
Petticoats (3)	1.75 [17]
Nightgowns (3)	2.40 [17]
Hat, winter	1.00
Hat, straw [19]	1.00
Gloves	1.50
Sundries	3.00 [20]
Total	$58.15

[12] For clothing, furniture and furnishings, and miscellaneous items, the average family consumption for 92 localities is used for each city.

[13] The percentage changes in the prices of a woolen dress and a woolen skirt are estimated from a ratio of the percentage change in the price of a coat and suit. The National Industrial Conference Board declined to state what this ratio is.

[14] The percentage changes in the price of a cotton skirt, house dress, and apron are estimated from a ratio of the percentage change in the price of women's waists, men's negligee shirts, and muslin underwear. The National Industrial Conference Board declined to state what these ratios are.

[15] Shoes, repairs, and overshoes are combined in later reports.

[16] The percentage changes in the price of shoe repairs and overshoes are estimated from ratios of the percentage change in the price of shoes. The National Industrial Conference Board declined to state what these ratios are.

[17] The percentage changes in the price of corsets, petticoats, and nightgowns are estimated from ratios of the percentage change in the price of muslin underwear. The National Industrial Conference Board declined to state what these ratios are.

[18] Summer union suits and winter union suits are combined in later reports, 3 for 33⅓ cents and 3 for 75 cents.

[19] The winter hat and straw hat are combined in later reports.

[20] The amount allowed each time for sundries is an arbitrary amount, varying with the changes in the cost of the other items of women's clothing.

United States Bureau of Labor Statistics—Cont.

TABLE III

MEN'S CLOTHING

Article	Quantity Weighting
Overcoat	¼
Suit, winter	¼
Suit, light weight	¼
Suit, Palm Beach, mohair	¼
Overalls or work trousers	2
Shirts, cotton	6
Hat, felt	1
Hat, straw	1
Cap	1
Union suits, athletic	3
Union suits, winter	1
Union suits, light weight	1
Pajamas or nightshirts	2
Socks, cotton	12
Collars	6
Neckties	2
Shoes, low	1
Shoes, high	1½
Rubbers	½
Whole soles and heels	1
Half soles and heels	1

SIX-YEAR-OLD GIRL'S CLOTHING

Article	Quantity Weighting
Wool coat	¼
Serge for dress (1), yds.	¼
Gingham for dresses (6), yds.	18
Gingham for apron, (1) yds.	2½
Petticoat or bloomers, winter	1
Petticoat or princess slip, summer	2
Drawers or bloomers, summer	5
Nightgowns, summer	1
Nightgowns, winter	1
Underwaists, summer	2
Underwaists, winter	2
Vests, summer	3
Union suits, winter	2
Shoes, low	3
Shoes, high	2
Stockings, cotton	12
Rubbers	1

National Industrial Conference Board—Cont.

TABLE IV

MEN'S CLOTHING

Article	1914 price
Suit	$15.00
Overcoat	10.00
Heavy trousers	3.50 [21]
2 shirts	2.00
3 work shirts [22]	1.75
3 pairs overalls	2.25
Shoes and repair	9.00 [23]
8 pairs hose	1.20
5 sets underwear [24]	3.50
2 nightshirts	1.50 [25]
Collars and ties	1.50 [26]
Hats, caps, and gloves, 1 each	5.00 [27]
Sundries	2.50 [26]
Total	$58.70

[21] The percentage change in the price of trousers is estimated from a ratio of the percentage change in the price of men's suits. The National Industrial Conference Board declined to state what this ratio is.

[22] One at 75 cents and two at 50 cents.

[23] The percentage change in the price of shoe repairs is estimated from a ratio of the percentage change in the price of shoes. The National Industrial Conference Board declined to state what this ratio is.

[24] Three at 50 cents, and two at $1.00.

[25] The percentage change in the price of nightshirts is estimated from a ratio of the percentage change in the price of men's negligee shirts, and women's muslin underwear. The National Industrial Conference Board declined to state what this ratio is.

[26] The amounts allowed each time for collars and ties and for sundries are arbitrary amounts, varying with the changes in the cost of the other items of men's clothing.

[27] The percentage change in the price of caps is estimated from a ratio of the percentage change in the price of men's felt hats. The National Industrial Conference Board declined to state what this ratio is.

United States Bureau of Labor Statistics—Cont.

TWELVE-YEAR-OLD BOY'S CLOTHING

Article	Quantity Weighting
Overcoat or mackinaw	½
Suit, wool	1
Trousers, cotton	2
Trousers, wool	1
Shirts or blouses, cotton	5
Union suits, summer	3
Union suits, winter	2
Pajamas or nightshirts	2
Cap	2
Neckties	2
Stockings, cotton	12
Shoes, high	5
Half soles and heels	5

b. Rent:

The number of quotations on dwellings in each city, which varies from 400 to 2,000 according to the size of the city, weights the prices within this group for each city, and for the United States. The percentage change in rent for the United States is computed from the aggregate rents paid in all of the 32 cities.

c. Fuel and light:

Average quantity consumption per family per year in each city as determined from 1918–19 investigation. This applies to both anthracite and bituminous coal. to gas, wood, kerosene, and electricity.

d. Furniture and furnishings:

The method followed in the computation of the percentage changes in furniture and furnishings is the same as that described above under clothing. Below is given the list of items on each of which at least four quota-

National Industrial Conference Board—Cont.

b. Rent:

The cities from which replies are received are divided into population groups (500,000 and over; 250 to 500,000; 100 to 250,000; 50 to 100,000; and under 50,000) and ratios computed of the total population. The average percentage change in cost in each population group is then weighted by the proportionate ratio of this group to the total population.

c. Fuel and light:

Average percentage change in anthracite and bituminous coal [28] weighted by 3.7; percentage change in light [29] weighted by 1.9. To obtain the percentage change in gas, the cities are weighted according to population, as described above for rent.

d. Furniture and furnishings:

The percentage change in the price of furniture and furnishings is included in sundries, and is weighted by the relative amount allowed for this item in 1914 as compared with the other items included in the sun-

[28] For this average percentage change in the price of coal for the United States, the percentage changes in the prices of anthracite and bituminous coal are given equal weighting. From an early study made by the Board of domestic consumption of coal it was determined that based upon the population using, these two kinds of coal were used in equal proportion in the United States.

[29] The percentage change in artificial gas is taken to represent the changes in light except in November of each year, when the change in electricity cost is averaged with that of gas, and then weighted by 1.9.

United States Bureau of Labor Statistics—Cont.

tions are secured in each of the 32 cities by agents. The weighting for each article was determined from the 1918–19 investigation and represents the average annual family consumption.

Article	Quantity Weighting
Carpets, rugs, wool, yds.	3.9
Matting rugs, grass, sq. yds.	.5
Linoleum, sq. yds.	1.8
Chairs, living room, and dining, each	.8
Tables, library, dining, and kitchen, each	.2
Couches, sanitary or bed, each	.1
Dressers and chiffoniers, each	.1
Buffets	.1
Bedsteads	.2
Bed springs	.2
Mattresses	.3
Baby carriages	.2
Pillow cases	1.9
Sheets	1.3
Tablecloths	.3
Towels	2.8
Blankets, cotton and wool, each	.5
Comforts	.2
Cook stoves and heating, each	.3
Brooms	3.0
Sewing machines	.1
Refrigerators	.1

e. Miscellaneous items:

The method followed in the computation of the percentage changes in this group of items is the same as that described above under clothing. Below is given the list of items on each of which at least four quotations are secured in each of the 32 cities by agents. The weighting for each item is the approximate average family consumption.

Article	Quantity Weighting
Street car: Regular fare, adult	550.
Movies: First floor, week night, adult	70.
Newspaper: Daily on street	313.
Sunday on street	52.
Doctor: Office visit	7.
House visit	7.
Obstetrical case	.1

National Industrial Conference Board—Cont.

dries group. The individual items within the furniture and furnishings group have the same weighting, or 1, with the exception of the average change in the price of the five articles of cotton yard goods which is weighted by 3, and the change in the price of furniture, which is weighted by 2.

e. Miscellaneous items:

The percentage change in the price of each item in this group is weighted by the relative amount allowed for this item in 1914 as compared with the amounts allowed for the other items in this group. To obtain the percentage change in the price of car fares, the cities are weighted according to population, as described above for rent. Below is shown the sum allotted for 1914, per week, to each item within this group together with the weighting assigned to each article. These amounts and weightings are estimates made by the National Industrial Conference Board of the distribution of sundries in an average family budget with $1079 income per year in 1914 and are based on other investigations of the cost of living

United States Bureau of Labor Statistics—Cont.

Article	Quantity Weighting
Medicine:	
Calomel tablets, ½ grain, doz.	4.
Aspirin tablets, 5 grain, doz.	4.
Castor oil, oz.	8.
Quinine pills, doz.	4.
Standard prescription, liquid, 2 oz.	3.
Standard prescription, liquid, 4 oz.	3.
Standard prescription, capsule or pill, 3 gr. mixture, 1 doz.	8.
Hospital: Pay ward, week	.2
Dentist:	
Filling, usual charge	4.
Crown, usual charge	.5
Plate, full upper, usual charge	.05
Spectacles: Gold filled rims, flat spherical lens	.3
Single separate lens	.5
Laundry:	
Men's stiff collars	104.
Men's shirts, soft cuffs attached	52.
Sheets	104. } or
Turkish towels	104. } or
Flat work, pounds	208.
Cleaning supplies:	
Soap, small, cakes	30.
Laundry soap (6–12 oz.), cakes	60.
Soap powder (8–16 oz.), pkgs.	12.
Cleaning powder (14–18oz.), pkgs.	24.
Barber: Shave	25.
Hair cut	10.
Toilet articles and preparations:	
Toothbrush	3.
Toilet soap, cakes	30.
Shaving stick or cream, sticks or tubes	2.
Tooth powder or paste, cans or tubes	6.
Talcum powder, cans	8.
Vaseline (1–2 oz.), jar	1.
Telephone: Residence	¼.
Tobacco:	
Cigar	110.
Cigarettes, pkgs.	60.
Cigarette tobacco (1–2 oz.), pkgs.	30.
Pipe tobacco (1–2 oz.), pkgs.	20.
Plug tobacco (1–3½ oz.), cut	20.

National Industrial Conference Board—Cont.

and upon previous local studies made by the Board.

Item	Amount Per Week	Weight
Carfare	$ .25	5.88
Medical:		
Doctor, nurse, hospital, etc.	.35	8.24
Drugs, toilet soaps	.15	3.53
Reading, stationary, telephone, etc.	.60	14.12
Recreation	.45	10.59
Insurance	.50	11.76
Furniture, furnishings, supplies	.60	14.12
Church, charity, gifts	.50	11.76
Organization dues	.35	8.24
Candy, ice cream	.10	2.35
Tobacco	.40	9.41
Total	$4.25	100.00

IV. WEIGHTINGS USED FOR THE DIFFERENT GROUPS

For each of 32 cities and for these cities combined

The method followed for the determination of the percentage changes in the total cost of living in each of the 32 cities and for the United States is as follows:

As explained above, under clothing, after the average price of each individual article has been computed and this average price multiplied by the average family consumption of this item, a total aggregate cost is made for each group of items. The aggregate cost of each group

For a varying number of cities

The percentage change in each group of items in the current month as compared with the preceding month is applied to the percentage increase of the preceding month over July, 1914. This percentage increase for each group is then multiplied by the group weights listed below. These weights have been obtained by making an average of the proportions spent for these items as shown in the following previously con-

United States Bureau of Labor Statistics—Cont.

of items (as clothing, for instance) in the current month is then compared with the aggregate cost of this same group of items in the preceding month. A percentage change is computed as between the two months compared. This percentage change is then converted into an index number and applied to the index number which has been previously computed for the preceding month as compared with the base period. As the expenditure for any group is only a part of the total expenditure, this percentage change in each group must be multiplied by the relative importance of this item in the total expenditure. The weights applied to the percentage changes in each of the groups —food, clothing, rent, fuel and light, furniture and furnishings, and miscellaneous items—were determined for each of the 32 cities from the 1918–19 investigation and represent the relative amounts of the total expenditure in each city spent for these items.[30]

For the United States, these figures were obtained from the result of the same investigation and represent the relative expenditure in 92 localities.

For the computation of the percentage changes in the total cost of living in the United States, the following weights are applied to the percentage changes in the cost of each of the groups of items:

Group	Weighting
Food	38.2
Clothing	16.6
Rent	13.4
Furniture and furnishings	5.1
Fuel and light	5.3
Sundries	21.3

National Industrial Conference Board—Cont.

ducted cost of living studies: The survey made by the United States Bureau of Labor Statistics for the United States in 1901, for New York City in 1917, and for Philadelphia in 1917; by the United States Railroad Wage Commission in 1915; by the Dallas Wage Commission in 1917; by R. C. Chapin for New York City in 1907. The proportions in each study have been weighted according to the number of families covered in each survey. Below are given the weights thus obtained and which are used by the National Industrial Conference Board in the computation of the index number of the cost of living for the United States:[31]

Group	Weighting
Food	43.1
Clothing	13.2
Rent	17.7
Fuel and light	5.6
Sundries	20.4

The following tables contain the figures computed by the two organizations:

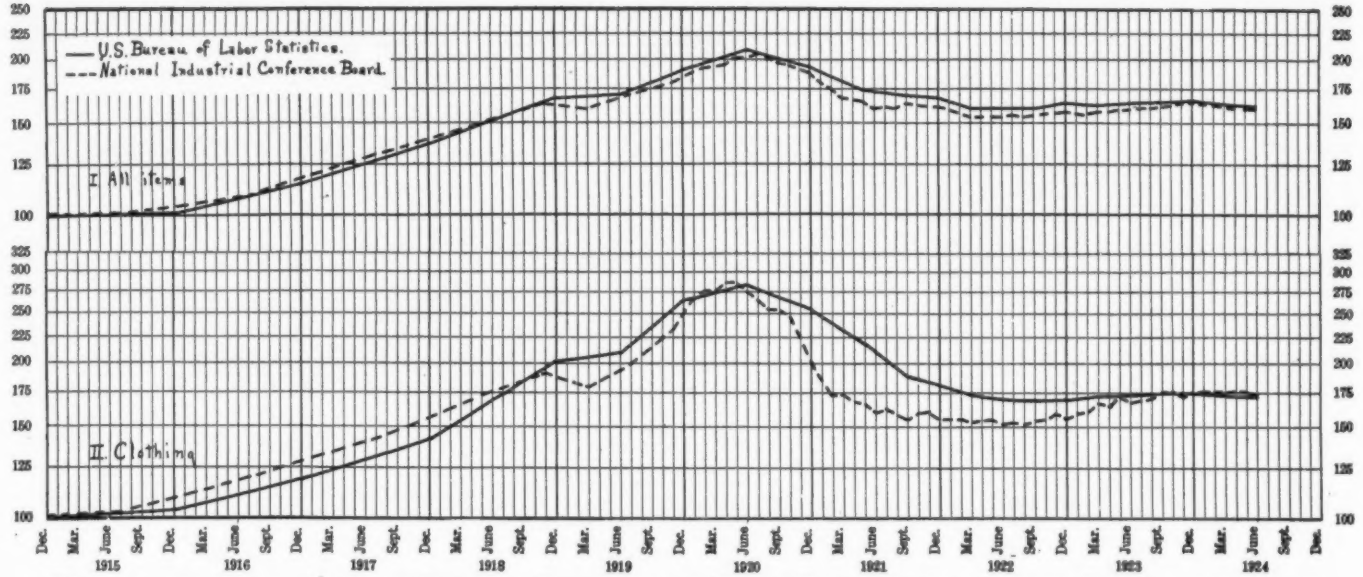
[30] The relative proportion of the expenditure for the different groups has been published for each city in the *Monthly Labor Review*, July, 1921, pp. 106–112.

[31] National Industrial Conference Board. *Research Report No. 9*, p. 4 and *Research Report No. 63*, p. 32.

TABLE V

INDEX NUMBERS OF THE COST OF LIVING AS SHOWN BY FIGURES OF THE UNITED STATES BUREAU OF LABOR STATISTICS AND THE NATIONAL INDUSTRIAL CONFERENCE BOARD

Date	All items		Clothing		Housing		Fuel and Light		Fur. & Furn.	Miscellaneous	
	U. S. B. L. S.	N. I. C. B.	U. S. B. L. S.	N. I. C. B.	U. S. B. L. S.	N. I. C. B.	U. S. B. L. S.	N. I. C. B.	U. S. B. L. S.	U. S. B. L. S.	N. I. C. B.
1914: July	...	100.0	...	100.0	...	100.0	...	100.0	...	...	100.0
Dec.	100.0	...	100.0	...	100.0	...	100.0	...	100.0	100.0	...
1915: July	...	100.5	...	103.0	...	100.0	...	102.0	...	...	100.0
Dec.	102.0	...	103.7	...	101.5	...	100.0	...	106.3	104.3	...
1916: July	...	108.7	...	120.0	...	101.5	...	104.0	...	...	104.0
Dec.	114.7	...	118.8	...	102.3	...	107.3	...	122.9	110.0	...
1917: July	...	131.3	...	143.0	...	105.0	...	126.0	...	...	117.0
Dec.	138.1	...	147.6	...	100.1	...	122.9	...	144.8	136.4	...
1918: June	...	152.2	...	177.0	...	115.0	...	135.0	...	...	150.0
Nov.	...	165.0	...	193.0	...	120.0	...	140.0	...	...	155.0
Dec.	169.0	...	203.3	...	109.2	...	146.4	...	205.4	161.0	...
1919: March	...	160.5	...	181.0	...	122.0	...	142.0	...	...	155.0
June	172.0	...	212.4	...	114.2	...	144.2	...	216.4	168.2	...
July	...	172.2	...	200.0	...	128.0	...	142.0	...	...	163.0
Nov.	...	182.2	...	235.0	...	138.0	...	148.0	...	...	175.0
Dec.	193.2	...	266.0	...	125.3	...	155.2	...	253.4	184.7	...
1920: Jan.	...	190.2	...	270.0	...	143.0	...	149.0	...	...	177.0
Feb.	...	193.5	...	277.0	...	145.0	...	149.0	...	...	178.0
March	...	194.8	...	277.0	...	149.0	...	149.0	...	...	183.0
April	...	196.6	...	288.0	...	150.0	...	151.0	...	...	183.0
May	...	201.6	...	287.0	...	151.0	...	155.0	...	...	183.0
June	210.2	203.0	284.7	276.0	134.9	151.0	170.2	161.0	281.4	195.5	185.0
July	...	204.5	...	266.0	...	158.0	...	166.0	...	...	185.0
Aug.	...	203.2	...	255.0	...	158.0	...	169.0	...	...	185.0
Sept.	...	199.4	...	255.0	...	159.0	...	178.0	...	...	188.0
Oct.	...	197.3	...	248.0	...	159.0	...	183.0	...	...	190.0
Nov.	...	193.1	...	228.0	...	166.0	...	200.0	...	...	192.0
Dec.	194.7	190.0	255.9	205.0	151.1	166.0	193.0	200.0	274.4	202.1	192.0
1921: Jan.	...	181.2	...	187.0	...	166.0	...	200.0	...	...	192.0
Feb.	...	176.3	...	174.0	...	166.0	...	198.0	...	...	190.0
March	...	168.7	...	174.0	...	171.0	...	187.0	...	...	185.0
Apr.	...	167.6	...	169.0	...	171.0	...	179.0	...	...	185.0
May	175.4	165.7	220.4	168.0	159.0	171.0	179.8	178.0	238.2	202.7	185.0
June	...	161.9	...	162.0	...	171.0	...	178.0	...	...	185.0
July	...	163.1	...	164.0	...	169.0	...	179.0	...	...	185.0
Aug.	...	162.0	...	159.0	...	169.0	...	179.0	...	...	183.0
Sept.	172.3	164.8	190.2	157.0	160.0	169.0	178.9	179.0	216.1	201.7	183.0
Oct.	...	163.7	...	160.0	...	169.0	...	179.0	...	...	180.0
Nov.	...	163.0	...	161.0	...	169.0	...	179.0	...	...	178.0
Dec.	169.5	162.7	182.6	157.0	161.4	169.0	179.3	179.0	209.6	200.8	178.0
1922: Jan.	...	161.4	...	156.0	...	169.0	...	178.0	...	...	178.0
Feb.	...	157.7	...	156.0	...	169.0	...	177.0	...	...	177.0
Mar.	162.4	154.7	173.8	154.0	160.9	165.0	174.1	174.0	198.3	197.4	174.0
Apr.	...	154.8	...	155.0	...	165.0	...	174.0	...	...	174.0
May	...	154.9	...	156.0	...	165.0	...	174.0	...	...	174.0
June	162.1	155.4	170.6	153.0	160.9	165.0	172.5	174.0	195.1	195.6	174.0
July	...	155.6	...	154.0	...	165.0	...	174.0	...	...	172.0
Aug.	...	154.5	...	153.0	...	165.0	...	181.0	...	...	172.0
Sept.	161.9	155.6	169.6	155.0	161.1	165.0	181.8	187.0	195.1	195.2	172.0
Oct.	...	157.1	...	157.0	...	165.0	...	187.0	...	...	172.0
Nov.	...	158.4	...	160.0	...	167.0	...	186.0	...	...	171.0
Dec.	164.8	158.9	169.8	156.0	161.9	167.0	184.6	187.0	200.2	194.7	171.0
1923: Jan.	...	158.1	...	160.0	...	167.0	...	187.0	...	...	171.0
Feb.	...	157.5	...	162.0	...	167.0	...	187.0	...	...	171.0
Mar.	164.1	159.2	172.7	168.0	162.4	170.0	184.4	186.0	209.0	194.5	173.0
Apr.	...	159.1	...	167.0	...	170.0	...	180.0	...	...	173.0
May	...	160.3	...	174.0	...	172.0	...	178.0	...	...	173.0
June	165.1	160.1	173.2	169.0	163.4	172.0	178.8	178.0	213.7	194.5	173.0
July	...	161.9	...	170.0	...	175.0	...	176.0	...	...	173.0
Aug.	...	161.6	...	171.0	...	175.0	...	176.0	...	...	173.0
Sept.	167.4	163.4	174.8	175.0	164.4	175.0	179.5	176.0	213.8	195.2	173.0
Oct.	...	164.1	...	176.0	...	175.0	...	178.0	...	...	173.0
Nov.	...	165.3	...	174.0	...	180.0	...	176.0	...	...	174.0
Dec.	168.4	165.0	174.6	175.0	166.5	180.0	182.2	176.0	213.8	195.8	174.0
1924: Jan.	...	164.6	...	176.0	...	180.0	...	175.0	...	...	174.0
Feb.	...	163.9	...	177.0	...	180.0	...	175.0	...	...	174.0
Mar.	165.4	163.2	174.2	176.0	167.0	185.0	180.5	172.0	212.8	195.2	174.0
Apr.	...	161.8	...	177.0	...	185.0	...	168.0	...	...	174.0
May	...	161.4	...	176.0	...	185.0	...	165.0	...	...	174.0
June	164.2	161.7	172.5	174.0	168.0	185.0	175.0	165.0	207.7	195.2	174.0



TREND IN THE COST OF LIVING AS SHOWN BY FIGURES OF THE UNITED STATES BUREAU OF LABOR STATISTICS AND OF THE NATIONAL INDUSTRIAL CONFERENCE BOARD, 1914 TO JUNE, 1924

[For figures of the United States Bureau of Labor Statistics, December, 1914 = 100; for those of the National Industrial Conference Board, July, 1914 = 100.]

TOTAL COST OF LIVING

Chart I shows that the trend in the total cost of living is very nearly the same in the two sets of figures.[32] During periods of rising prices and since 1922, when prices have been fairly stable, the two curves are very close to each other. During periods of falling prices, however, as in 1919 and 1921, the figures of the National Industrial Conference Board show a greater decline than those of the United States Bureau of Labor Statistics. The drop in the figures of the National Industrial Conference Board for 1919 probably registers some of the decline which the figures of the United States Bureau of Labor Statistics would show if these figures had been taken quarterly as they are now.

The first investigation of changes in the cost of living by the National Industrial Conference Board was made in June, 1918, and the next in November, 1918. Since this time, "large" investigations have been made each year in March, July, and November. Below is a table which shows how the number of reports received and the number of cities from which these reports have been sent have changed from period to period.

TABLE VI

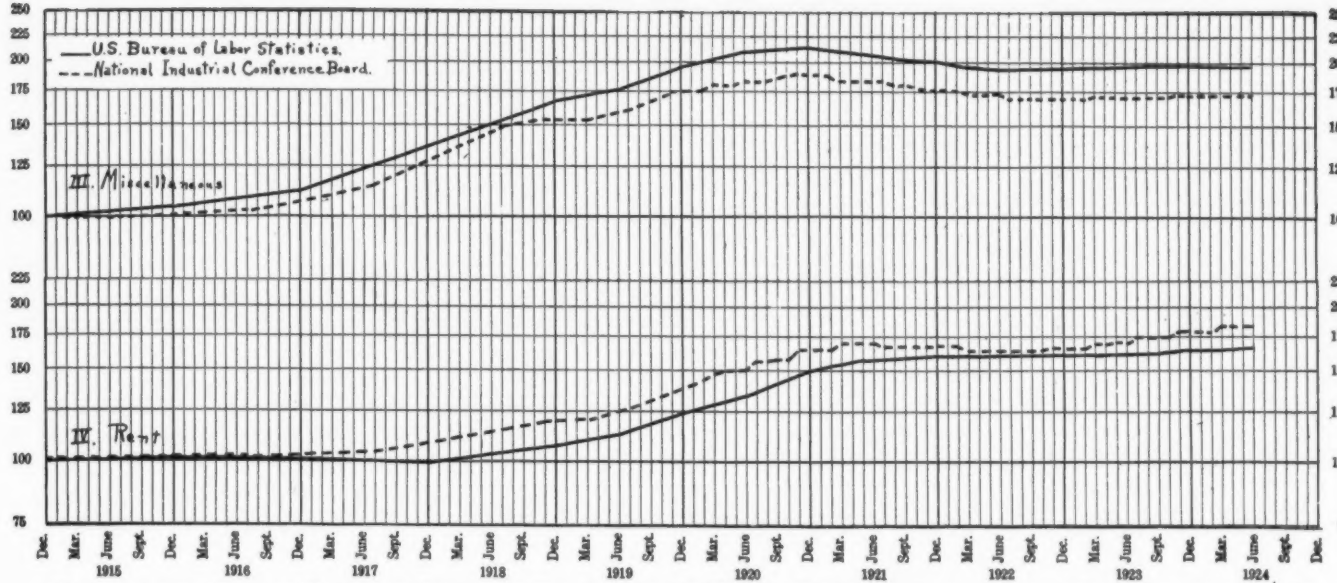
NUMBER OF REPORTS AND NUMBER OF CITIES REPRESENTED FOR CLOTHING AND RENT BY THE REPORTS OF THE NATIONAL INDUSTRIAL CONFERENCE BOARD

Date of investigation	Clothing		Rent		Date of investigation	Clothing		Rent	
	Number of reports	Number of cities	Number of reports	Number of cities		Number of reports	Number of cities	Number of reports	Number of cities
June, 1918	46	32	128	79	Mar., 1921 ...	200	74	479	165
Nov., 1918	112	46	197	93	July, 1921 ...	191	78	467	167
Mar., 1919	152	46	236	114	Nov., 1921 ...	196	77	490	164
July, 1919	146	43	317	155	Mar., 1922 ...	189	70	476	165
Nov., 1919	195	66	331	156	July, 1922 ...	186	75	481	168
Mar., 1920	208	72	355	156	Nov., 1922 ...	186	74	471	164
July, 1920	203	75	359	158	Mar., 1923 ...	198	87	470	164
Nov., 1920	166	67	461	167	July, 1923 ...	183	78	448	161

For the other months of the year, the National Industrial Conference Board makes estimates of changes in prices from the reports of a much smaller number of firms.[33]

[32] Because the first prices of the United States Bureau of Labor Statistics were secured for December, 1914, in order to use the year 1913 as a base it was necessary for the Bureau to estimate the increase from 1913 to December, 1914, from retail prices of food, and wholesale prices of other articles, already collected by the Bureau (*Monthly Labor Review*, Jan., 1920, p. 97). This method probably shows the approximate changes, however, as there was very little change in prices from 1913 to 1914. As the figures of the National Industrial Conference Board are based upon prices in July, 1914, for comparative purposes of this article, the figures of the United States Bureau of Labor Statistics have been converted from 1913 to December, 1914, as a base, and any slight inaccuracy that there may be in these figures from 1913 to 1914 has therefore been eliminated.

[33] In January, February, and April, 1924, the number of these reports was as follows: Rent: Jan., 69; Feb., 68; April, 60. Fuel: Jan., 44; Feb., 42; April, 42. Clothing: Jan., 45; Feb., 49; April, 51.


U.S. Bureau of Labor Statistics.
National Industrial Conference Board.
III. Miscellaneous
IV. Rent
250
225
200
175
150
125
100
75
Dec. Mar. June Sept. Dec. Mar. June Sept. Dec. Mar. June Sept. Dec. Mar. June Sept. Dec. Mar. June Sept. Dec. Mar. June Sept. Dec. Mar. June Sept. Dec. Mar. June Sept. Dec. Mar. June Sept. Dec. Mar. June Sept. Dec. Mar. June Sept. Dec.
1915 1916 1917 1918 1919 1920 1921 1922 1923 1924

TREND IN THE COST OF LIVING AS SHOWN BY FIGURES OF THE UNITED STATES BUREAU OF LABOR STATISTICS AND OF THE NATIONAL INDUSTRIAL CONFERENCE BOARD, 1914 TO JUNE, 1924

[For figures of the United States Bureau of Labor Statistics, December, 1914=100; for those of the National Industrial Conference Board, July, 1914=100.]

The figures of the United States Bureau of Labor Statistics, showing the changes in the cost of living in the United States are based on retail prices in the same 19 cities from December, 1914 to December, 1917. Beginning with December, 1917, to these original 19 cities were added 13 others. So far as possible, prices are secured each time from the same dealers in these same 32 cities. In the quotations from the individual dealers, identical brands or grades of identical articles are followed. Whenever a new dealer is added, prices are secured not only for the current month, but also for the preceding date. If a dealer does not report for the current month, his prices are not included in the average for the preceding month when compared with the present month. This is also true of articles and brands, for when it is impossible to carry longer a particular item or a particular brand of article, the price of this item is eliminated for the preceding month. The price of the article or brand substituted is used in both the preceding and current month. Hence, only identical brands of the same articles from identical dealers are used.

That the curves plotted showing the trend in the cost of all articles are approximately the same may be due to the fact that the National Industrial Conference Board uses the food figures as computed by the United States Bureau of Labor Statistics and to the fact that in addition there is similarity in the figures showing changes in the cost of fuel and light. The use of the food figures of the United States Bureau of Labor Statistics and the similarity in the fuel and light figures eliminates chances for error in about 49 per cent of the budget. It also follows that, except in periods of declining prices, the National Industrial Conference Board's higher rent figures and its lower figures for sundries and clothing must equal in the aggregate the United States Bureau of Labor Statistics' lower rent and higher sundries and clothing figures. In periods of declining prices, it is evidently the figures for sundries and, in particular, the figures for clothing of the National Industrial Conference Board that cause a greater decline in the figures of this Board than in those of the United States Bureau of Labor Statistics.

CLOTHING

The greatest difference between the figures of these two organizations is in clothing. Those of the National Industrial Conference Board for 1921 produce not only a drop far below the figures of the United States Bureau of Labor Statistics but also show wobbles between months in which more quotations are secured. For example, between November, 1920, and March, 1921, March and July, 1921, and July and November, 1921, there are sags in the curve. This probably would not have

been the result if the figures for these "between" months had been based on more quotations. The reason for the big difference in the trend in clothing prices is difficult to determine. The following may be some of the reasons:

(1) The fact that the National Industrial Conference Board does not get prices on 11 articles out of the 22 articles on which the woman's budget is based, but estimates the changes in prices of these 11 articles from the changes in prices of the 11 articles on which they do get prices. (See Table II.) Therefore, instead of the budget being based on the trend in the 22 items of women's apparel listed in the budget, it is, as a matter of fact, based only on actual changes in 11 of these and on assumed changes in the other 11. The following are the only items on which actual prices are secured: Coats, suits, cotton waists, shoes, hosiery, summer union suits, winter union suits, muslin underwear, winter hat, straw hat, and gloves. The same criticism may be made with respect to the men's clothing in that actual prices are secured for only 9 of the 17 articles listed in the budget. For sundries and for collars and ties, as in the case of sundries for women's clothing, the variations of the National Industrial Conference Board are based on purely arbitrary amounts which are estimates derived from the changes in the cost of all the other items of clothing in each respective budget. For the items in the men's budget as well as for the items in the women's budget on which prices are not secured but the price changes for which are estimated from a ratio of the changes in other items (see Tables II and IV), the accuracy of the price changes is not only doubtful but impossible to verify. As the National Industrial Conference Board declines to state in any case what this ratio is, saying that they consider these processes as belonging to them just as much as any other mechanical methods of computation, it is impossible to check the changes given to these articles by any other figures. It is, moreover, impossible in most cases to check the published reports of the National Industrial Conference Board and compute the percentage changes that have been applied to the individual items upon which actual prices are not secured. For example, as is generally known, the cost of shoe repairing has not declined from the peak prices nearly as much as has the cost of shoes. The published figures of the National Industrial Conference Board, however, give the combined cost of shoes and shoe repairing, and, in the women's budget, overshoes are also included, making it impossible to compute the percentage allotted to each item. In other cases, where the change in the cost of one item is based on a ratio of the changes in two or three other items, it is impossible to check the figures because of not having this ratio.

(2) The National Industrial Conference Board does not include in the computation of its cost of living figures changes in the cost of children's clothing, but it assumes that children's clothing fluctuates in the same way that clothing for adults does. The United States Bureau of Labor Statistics computes changes in the cost of clothing not only for a man and woman but also for a boy of twelve years and a girl of six. Separating the cost of the clothing in 10 cities for each of these and combining to find the cost of adults' clothing and of children's clothing, the figures show that children's clothing does not fluctuate as does adults' clothing. The following table demonstrates this:

TABLE VII

TREND IN THE COST OF CLOTHING AS SHOWN BY FIGURES OF THE UNITED STATES BUREAU OF LABOR STATISTICS FOR 10 CITIES,[84] JUNE, 1920, to JUNE, 1924

[June, 1920=100]

Date of investigation	Adults' clothing	Children's clothing	Date of investigation	Adults' clothing	Children's clothing
June, 1920	100	100	Dec., 1922	58	61
Dec., 1920	89	91	Mar., 1923	59	62
May, 1921	76	78	June, 1923	59	63
Sept., 1921	65	69	Sept., 1923	60	63
Dec., 1921	62	66	Dec., 1923	60	63
Mar., 1922	59	63	Mar., 1924	60	63
June, 1922	58	62	June, 1924	59	63
Sept., 1922	58	61			

As it is generally believed and as the figures for the ten cities prove, in periods of declining prices, children's clothing does not decline nearly so much as does adults' clothing. It is in periods of declining prices that there is the largest difference between the figures of the National Industrial Conference Board and of the United States Bureau of Labor Statistics. Some of this discrepancy in the figures of the National Industrial Conference Board seems to be due to the fact that it does not include in its figures price changes of children's clothing.

(3) We have seen that the National Industrial Conference Board's figures are based upon a variation in price of a very few articles of clothing. This in itself is bad, but the further factor of the method of obtaining the figures also lends itself to inaccuracies. The Board obtains practically all of its data through questionnaires or correspondence. This method cannot be as accurate as the method followed by the United States Bureau of Labor Statistics of personal contact through trained agents who make it a point to see that the data collected from period to period apply to the same brand and grade of

[84] Atlanta, Baltimore, Birmingham, Boston, Buffalo, New York, Chicago, Denver, Los Angeles, and Seattle.

articles. In other words, the questionnaire has more of an opportunity for discrepancies because of the fact that the dealer filling it out does not have in mind the exact articles on which he quoted prices in the former questionnaire. The National Industrial Conference Board does endeavor to get prices from time to time on the same grade of certain items by stating on the questionnaire that this is desired and also by typing in on the questionnaire the prices of the items in 1914 and the prices last reported. The Bureau of Labor Statistics has, however, found it an exceedingly difficult thing to substitute equivalent grades and brands when this is necessary even with an agent in the store to look at the goods and to consult with the dealer relative thereto. Even though these selections may represent the personal opinion of the agent after consultation with the dealer and with other dealers in the same and other cities, the Bureau feels that this personal opinion of about a dozen agents, thoroughly trained in their work, familiar with the details both in the field and office, is not to be compared with the personal bias or misunderstanding that would exist if questionnaires were used and each of the thousands of dealers was free to enter his own prices. Moreover, when some article is no longer used another which has become generally used for the same purpose must be substituted. It is difficult to see how a correct idea could be obtained of this change in consumptive demand without personal contact of agents. In addition, agents are not sent to the same cities on consecutive dates and in this way the work of one agent is checked by that of the next.

(4) Another reason for the figures on clothing of the National Industrial Conference Board being so much lower than those of the United States Bureau of Labor Statistics in periods of rapidly declining prices is perhaps due to the fact that the National Industrial Conference Board computes its averages from a varying number of reports each time. These reports not only vary within the cities, but the cities from which the reports come are not the same. In November, 1920, the date of the first big drop, the number of reports on clothing fell from 203 to 166 and the number of cities represented by these reports decreased from 75 to 67. On this date, 5 eastern cities and 7 middle western and southern cities were dropped. Two new eastern cities and 2 new middle western cities were added. There was a reduction of 11 quotations in the east and 26 quotations in the other parts of the country, 7 of these being from the west, while 5 new quotations were added in the east and 8 for the middle west and south. If clothing prices did not decrease so much in the middle west, west and south as in the east, the dropping of twice as many quotations for these sections as for the east would tend to let

the average be unduly pulled down. Taking the decrease in clothing as published by the United States Bureau of Labor Statistics for 32 cities,[35] and separating these cities into different geographical sections, the average decrease from June, 1920, to December, 1920, was 10.3 per cent in the east; 10.7 per cent in the middle west; 10.5 per cent in the south; but only 8.0 per cent in the west.[36] Two new quotations were added for the first time in Kansas City, and by the figures of the United States Bureau of Labor Statistics this city from June to December showed a decrease of 14 per cent. In Minneapolis, one quotation was dropped and this city showed a decrease from June to December of 7 per cent. This shows that the bringing in or dropping of cities and the variation in the number of replies from the cities from period to period could easily make a difference show in the figures that might be due largely to this reason and not to a change in prices if identical cities had been kept and if identical firms had reported on identical brands and grades of the same articles. Even though the figure is for the United States as a whole, the figures are likely to be misleading when all the prices are lumped together no matter how much the number of reports and the number and location of the cities represented fluctuates from one time to the next.

MISCELLANEOUS ITEMS

For miscellaneous items,[37] there may be many reasons for the trend in the figures of the National Industrial Conference Board being lower than those of the United States Bureau of Labor Statistics. (1) It would be very hard for merchants to follow identical items on the articles listed on the questionnaires for furniture and furnishings sent out by the National Industrial Conference Board. The groups are too large—individual articles should be specified under the general groups. To ask a dealer to estimate what is the change in the price of furniture is not as accurate as having him report on certain definite articles of furniture. With any of these groups, such as furniture or crockery and glassware, for instance, certain articles may be in mind at one time and certain other articles at another time. (2) Another thing that may keep this group down is that church contributions, organization dues, insurance rates, and expenses for gifts may not decrease with

[35] *Monthly Labor Review*, Feb., 1922, pp. 58–63.

[36] These cities have been grouped as follows: East—Baltimore, Boston, Buffalo, New York, Philadelphia, Portland, Me., Washington, Scranton. Middle west—Chicago, Cleveland, Detroit, Cincinnati, Indianapolis, Kansas City, Minneapolis, Pittsburgh, St. Louis. South—Houston, Jacksonville, Mobile, Norfolk, Savannah, Atlanta, Birmingham, Memphis, New Orleans, Richmond. West—Los Angeles, Portland, Ore., San Francisco, Seattle, Denver.

[37] In the chart for miscellaneous, the figures of the United States Bureau of Labor Statistics for furniture and furnishings have been included in the miscellaneous items by combining them in a weighted average.

the decrease in the general cost of living. The idea of the National Industrial Conference Board is that by applying to these items the change in the other items of the cost of living, the same protection or benefit is given as formerly. (3) The weighting for insurance is 11.76, for church, charity and gifts, 11.76, for organization dues, 8.24, making a total weighting of about 32 out of the total for this group of 100. This seems a large proportionate weighting out of the group not to be based upon actual prices. (4) The less increase in the figures of the National Industrial Conference Board may also be partly accounted for by the fact that changes in the cost of dental work, spectacles, laundry, cleaning supplies, barber, toilet articles, with the exception of toilet soap, and telephone are not provided for. All the items included in the budget of the National Industrial Conference Board for miscellaneous are included in that of the United States Bureau of Labor Statistics with the exception of insurance, church and charitable contributions and organization dues. If these items were included in the figures of the United States Bureau of Labor Statistics and increased by more or decreased by less than the change in the general cost of living, as stated by the National Industrial Conference Board as the way in which their figures are computed for these items, the discrepancy between the two sets of figures would be greater than at present, for the figures of the United States Bureau of Labor Statistics would be further increased. (5) Up until December, 1918, the United States Bureau of Labor Statistics assumed that the increase in the cost of miscellaneous items was the same as that of the other groups of items. This may have given a higher figure for this group than the actual prices would have justified.

RENT

The increase in rents as shown by the National Industrial Conference Board has been, since about 1917, above that as shown by the figures of the United States Bureau of Labor Statistics, the greatest differences being between 1918 and 1921. While the National Industrial Conference Board has received reports from many more cities than has the United States Bureau of Labor Statistics, the number of reports to the National Industrial Conference Board has varied each time. Not only the number for the individual cities has varied, but there has been a good deal of variation in the cities included; that is, a report would be received from a city and perhaps at the next time no reply would be received from this city, but replies would be received from other cities in other sections of the country not reporting on the previous date. Then, too, for many cities, the estimate of the change

in the price of rents has been made by the National Industrial Conference Board from only one reply. While it is true that this one reply has come in many cases from real estate boards which would have a knowledge of many houses, it is probably in many cases the estimation of the secretary of the board, or the opinion of one man. Also, by simply reporting a generalized estimate of rent prices rather than actual prices on certain selected houses each time, there may be a tendency to remember only those houses where changes have occurred.

The figures for rent of the United States Bureau of Labor Statistics, on the other hand, are taken by agents of the Bureau off of the books of real estate agents and owners and are on identical houses from time to time, no matter if there has been a change in the agent, owner, or tenant. Up until May, 1921, when a dwelling changed hands, effort was not made by the Bureau to follow up this house, but from this time for about a year, the agents were instructed to follow up all dwellings and secure the prices on these dwellings that had changed agents or owners. This, however, entailed so much additional work and the funds of the Bureau being limited, it was necessary to direct the agents to follow up only blocks of houses or apartment houses containing several apartments. It has been impossible to follow individual private houses, when these have changed hands. Of course, there are comparatively few of these, but the elimination of them may to some extent keep the figures for rent of the United States Bureau of Labor Statistics lower than they would be if the funds of the Bureau permitted each house being followed when it changed owners or agents.

It would seem that the method employed by the United States Bureau of Labor Statistics of getting actual prices on the same houses each time ought to be a much more accurate way to determine rent changes. It would also seem that the number of houses on which quotations are secured—from 400 to 2,000 in each city, 400 in cities the size of Birmingham, Ala., to 1,200 in Philadelphia, 1,500 in Chicago, and slightly over 2,000 in New York, making a total of over 23,000 dwellings (23,657 in June, 1924), ought to be a sufficient number, except perhaps in New York, to fairly show the trend.

As the housing conditions have changed, the Bureau has endeavored to drop out the type of house being abandoned and to put in the types of houses which were being rented. The criticism that the Bureau was slower than it should have been in making these modifications, and for this reason, the increase in rents had not shown as great an advance as it should have, may, perhaps, be partly justified. At the present time, however, the Bureau feels that this revision has been

fairly generally made and the type of houses now included is quite representative of the houses lived in by working people.

GROUP WEIGHTINGS

The proportionate weightings of the United States Bureau of Labor Statistics were made from the investigation of 1918–19. The criticism has been made that this period was not a good period in which to secure figures from which to compute basic weights. While it is true that workers were receiving more money as wages than in 1914, yet the rate per week or per day was not increased as fast as the cost of living increased, and the purchasing power of the rate of wage paid per week or per day measured by the cost of living shows that the real wage was much less from 1917 to 1919, inclusive, than it had been in 1913 and 1914, and much less than it became in 1920, 1921, 1922, or 1923, when the peak was reached. It would, therefore, not be true that the workers had relatively more money to spend on luxuries and unnecessaries than they had in 1914 because a relatively larger percentage had to be spent on the necessary items of the cost of living.[33]

It has also been suggested that because the Bureau of Labor Statistics' weightings are the result of the investigation of 1918–19 when clothing prices had risen more than had some of the other items such as rent that too large a weighting is given to clothing and too small a proportion allowed for rent by the Bureau. The heavy weighting of a group would work both ways. In a period of rising prices, it would cause the total cost of living to rise more than it would with a smaller weighting applied to the group, but in a period of declining prices, it would cause a greater decrease to be shown in the figures than if a lesser weighting were used. If the other figures of the National Industrial Conference Board and of the United States Bureau of Labor Statistics were alike, and if the percentage increases in clothing, for example, were the same, but these percentage increases were weighted by 17 for the budget of the Bureau and by 13 for the Board's budget, during periods of declining prices, the two curves for the total cost of living would be brought nearer together than in periods of rising prices. The heavier weighting for clothing of the United States Bureau of Labor Statistics would not, therefore, account for the difference in the curves in periods of declining prices. These group weightings, moreover, affect only the trend in the total cost of living and have no effect upon the trend in the individual groups.

[33] Based on 1913 as 100, and measured by the increase in the cost of living, the index numbers representing the purchasing power of rates of wages paid per week were: 1914, 99; 1915, 97; 1916, 90; 1917, 79; 1918, 75; 1919, 79; 1920, 91; 1921, 109; 1922, 109; 1923, 116. (United States Bureau of Labor Statistics, Bul. No. 325, p. 19, and *Monthly Labor Review*, Feb., 1924, p. 94.)

The weightings for the total budget used by the National Industrial Conference Board are averages of cost of living surveys made in 1901, 1907, 1915, and 1917 (three studies in this year), weighted by the number of families covered. Three of the studies were made before we entered the war and three were made afterward, the greatest weight being exerted by the 1901 study of 16 years before. It cannot either then be said to represent strictly a pre-war budget expenditure, for the increase in the cost of living was noticeable as early as 1916.

And neither budget perhaps represents exactly conditions of today. To determine what are the proportionate expenditures for the different items in a budget of today, another investigation would have to be made. It is the opinion of the writer that we have gotten from investigation of large numbers of families about all that we can. While these investigations have shown slight variations at different times in the proportion spent for the different groups of items such as food, clothing, and rent, the difference has not been so vital as to materially affect the final result. The next progressive step in budget studies should be made from the standpoint of health. The health of the workers' families should be graded, and the diets of these higher rated families should be studied and a budget prepared therefrom.[39] This would afford a better basis than to simply take the average amount of certain articles consumed by a large number of families without respect to whether or not these families are being properly nourished. When it comes to the selection of the proper number and proportion of other items than food, that would be necessary for health, the task would be more difficult, but it does not seem insurmountable.

The discrepancy in the figures of the two organizations does not seem to be due to the weightings used, but to a difference in basic prices. For the collection of basic prices, which are comparable from month to month, the methods employed by the United States Bureau of Labor Statistics of securing prices on identical grades of the same item from the same dealers from time to time are far superior to those of the National Industrial Conference Board. An index number based upon these prices should, therefore, be much more reliable.

[39] See also "A Suggestion for Determining a Living Wage" by Dorothea Davis Kittredge in the *American Economic Review*, Vol. 13, p. 225.

NOTES

THREE YEARS' PROGRESS IN CURRENT BUSINESS STATISTICS.

By Mortimer B. Lane, *Bureau of the Census*

The obvious need for facts and still more facts on the current business situation sometimes blinds us to the really significant increase in the number of such facts that have been made available recently to the general public, and to the ease with which the public can secure these items all under one cover and with accuracy and promptness. I refer especially to the collection of statistics organized by the Department of Commerce just three years ago, and published in their *Survey of Current Business*, because the growth of this publication has been a measure of the growth and of the understanding of current business statistics.

If we look back to conditions existing in the early part of 1921, we are struck by the groping around by business men for some straws which would tell which way the business wind was blowing. The sudden cessation of the war had put before business the problem of transferring to a peace basis but the basis was entirely different from the conditions existing before the war broke out. The enormous fluctuations in all kinds of business and industry from 1919 to 1921 testify to the lack of a basis of fact on which to build definite programs and policies. The result was, first, a depression, then a great demand for the war-forbidden luxuries and the carrying away of all but the most rugged supports in the mad rush to supply this demand and to stock up to meet its unexpected continuance, while finally the collapse of the wave washed away the illusory profits and left business deserted and without a compass in uncharted seas.

During the war, a rather complete statistical service had been built up hurriedly to serve the pressing needs of industrial mobilization. The publication of some of these data was continued thereafter and found of the utmost benefit to the industries concerned. But on the general business situation there were very few guideposts. By getting some figures here and some there and a smattering of opinions, some sort of idea of the situation might be developed, but it had to be rather hazy. A few organizations had made a beginning in fact gathering, notably the Federal Reserve Board, which had collected and published figures on the leading industries with comparisons for several

months, and the Harvard Bureau of Economic Research, whose compilations were on a broader base but not so available to the general public.

A NEW DEPARTURE IN STATISTICAL PRESENTATION

When Secretary Hoover took charge of the Department of Commerce in 1921, he instituted an aggressive campaign to give the business man the facts so urgently needed and in a form that could be readily grasped and understood. Mr. Hoover as an executive needed these data for his own information in guiding the Department of Commerce and he believed that the publication of these figures in a clear manner would be of considerable help to business men. No duplication of existing figures was to be made, merely the collection of data already in existence and the addition of new data only when not elsewhere available. There was no interference with or attempt to discredit the statistics put out by other organizations, governmental or private, as once in a while is asserted, for not only were the existing figures retained and not duplicated, but they were given much wider publicity than before and with full credit to the compiling organizations, a matter in which other agencies were not so careful. This undertaking was not to destroy existing statistical compilations but to build them up into a composite whole which would be comprehensive and yet readable and easily grasped by the business executive.

At first, a small edition was prepared from the data which could be assembled within a short time and this was sent to executives and publicists throughout the country for their opinions. The response was overwhelming in both volume of replies, and the unstinted approval which they expressed. An official of one of our largest rubber companies called it "the greatest step in our industrial life since the inauguration of the Federal Reserve System" while the sales manager of one of our great automobile companies predicted that it would soon become "the right hand of modern business," to quote only two of the hundreds of appreciative and sympathetically critical letters.

The *Survey of Current Business*, launched thus auspiciously, differed from any previous publication in several ways.

First, it brought together more current data in our industrial and commercial life than had ever before been put under one cover. As early as the third issue, about 500 separate items were included, covering the various phases of business in wool, cotton, cotton goods, silk, iron and steel, non-ferrous metals, coal and coke, petroleum, paper, rubber, building construction, lumber, brick, cement, hides and leather, chemicals, grains, meats, dairy products, fats and oils, tobacco, sugar,

water transportation, railroads, living costs, employment, foreign trade, retail trade, public finance, banking and securities markets. No other current publication carried as many as 200 items on general conditions.

Secondly, monthly figures were given for two years, enabling comparisons throughout the period.

Third, monthly averages were presented for as many years as possible as far back as 1913, so that a glance would show the current month's relation to the level of operations for any previous year.

Fourth, and probably the most advanced step, was the calculation of index or relative numbers for all items, using 1913 as the base wherever possible, on account of the general desire to compare with the latest pre-war year. These relative numbers, placed on a page opposite the numerical data, presented a comparative picture of the relative trends of production with stocks or with prices in the particular industry or with the same items in other industries.

Fifth, these 500 items were collected from compilations already in existence as collated by government organizations, by trade associations or by private firms or technical journals.. In some cases revisions had to be made in the source materials to secure comparable figures as far back as 1913 or a method of computation had to be used where the number of reporting firms varied each month, but the results tended to clarify the original data used.

Sixth, a complete statement of sources, of methods of computation and of other explanatory data was appended to each detailed table, thus giving the reader a true conception of the value and extent of the figures published and a reference for further details or explanations, while each compiler of statistics used was properly credited with the data.

Seventh, illustrations of the meaning of the tables were given by means of diagrams indicating the principal trends.

Eighth, through access to confidential reports for the Census of Manufactures, it was possible to show for most association reports the percentage of the industry covered by the total of all firms reporting data for each association's figures, and thus ascertain the representative character of these reports.

DIRECT COLLECTION OF STATISTICS

Great as was the advance over existing sources of the statistical information given in the early issues of the *Survey of Current Business*, a steady improvement was seen in the *Survey* itself as new issues were put out.

In the first place, it was soon recognized that the existing business statistics compiled by trade associations were incomplete in that only the membership of such associations was represented and this was often 50 per cent or less of the output of the particular industry. In order to secure greater completeness and make the figures really representative, the Bureau of the Census started to collect, in coöperation with certain associations, figures from non-member firms, whose totals were then added to the association figures. The automobile figures were the first to be collected and published monthly in this manner and they have given the industry almost complete production figures promptly each month. Gradually other industries were added, until now such data are collected directly by the Census Bureau in coöperation with associations in almost a dozen industries, and still others are in process.

A further extension of this movement has been the collection of data direct from all manufacturers where a trade association or individual firms suggested the need for the statistics but where no outside agency was in a position to take up the work. These statistics have also grown in volume and for two industries — boots and shoes, and flour milling — over 1,000 firms regularly report their monthly statistics to the Bureau of the Census for the monthly tabulation, and over a dozen other industries are covered by a smaller number of firms.

The presence in the Census Bureau of the confidential reports of manufacturers made for the biennial census of manufactures enabled its agents to secure correct lists for this work and to ascertain the proportion of the industry covered by reports sent in, and also to concentrate on the larger firms to secure more complete reports where necessary. By close coöperation in the office the monthly statistics have been tied up with the biennial census in a comparable manner and the information secured in one line has been made available for the other, thus reducing duplication and enabling the census lists to be brought up-to-date currently.

The industries and the public have been given the benefit of quick publicity on the results of these monthly inquiries, whether made in coöperation with associations or reported directly by individuals. Special release statements have been issued to the press and sent to the reporting firms as soon as the figures have been received and tabulated, and the data are later brought together with the other industrial items in the Survey of Current Business.

The confusion in the trade association field resulting from the various interpretations of proper legal activities of such associations along statistical lines caused several associations to give up their statistical work entirely, in spite of its usefulness. Others were unwilling to

sacrifice the benefits obtained from proper use of this information to the uncertainty which its loss would entail and they were able to find in the coöperation of the Bureau of the Census a source for the continuation of the collection of these monthly figures, which was at once authoritative, more representative and able to secure prompt and impartial distribution of the facts. Also, by virtue of its publication of the *Survey of Current Business*, the bureau acted as a disseminator of association statistics, even when not combined with individual data. Thus the *Survey of Current Business* became a vital factor in the proper solution of the question of trade association statistics.

MORE PROMPT DISSEMINATION

Not alone in these special inquiries was it made possible to secure prompt figures. It soon became evident that the time taken to compile and print a complete history of a particular month's business conditions would reach the reader too late for the most advantageous use of all this material. So a system of advance leaflets was inaugurated, which were sent free of charge to all subscribers to the *Survey of Current Business* who requested them, and gave a bare outline of the latest figures available. At first these were sent out twice a month, later they were issued three times a month, and now hardly a week goes by but what the subscriber to the *Survey of Current Business* receives his leaflet giving the statistics made available during the previous week, with a short summary text and a diagram of some important movement. Thus the reader gets his latest information right up to the minute and still has later in the *Survey* itself a complete history of the month as shown by business statistics.

The weekly advance leaflets and press statements have also helped to shatter the tradition that a certain group of current figures are the property of a particular bureau or agency, to be held unpublished by them until some report was put out, sometimes weeks later. Such figures were of no use to the business man, and this viewpoint has finally resulted in a number of government figures being made available much earlier than usual by the departments themselves and they have been given general distribution immediately thereafter in the advance leaflets of the *Survey of Current Business.*

INDEX NUMBERS

The widespread use of index numbers to measure business movements is reflected by the many such combined index numbers published in the *Survey of Current Business.* Wherever possible, existing index

numbers have been published, with due credit to their authors, but in a few cases the Department of Commerce has through the *Survey* developed its own index numbers in fields not yet explored or else not completely covered.

The production index numbers which the Department of Commerce has compiled are the most complete obtainable, covering approximately 44 per cent of the manufacturing output, 80 per cent of forest products marketed, 87 per cent of the mineral production, 94 per cent of the crop production, and about 99 per cent of marketed livestock and livestock products. These index numbers differ from those put out by the Federal Reserve Board in their greater completeness, as the latter organization aims to publish a more selective index number more promptly.

A new field was covered by the inauguration of index numbers on commodity stocks and on unfilled orders, both of which have been proved of wide interest and usefulness and which are looked forward to with such anticipation that they are now put out in special release statements each month.

Conferences have been arranged between the Department of Commerce, the Federal Reserve Board and the Department of Agriculture to coördinate the work on index numbers, prevent duplication and give each the benefit of the ideas of the others. Uniformity in the use of the best types of formulae, weightings, and other considerations has been inaugurated by these departments of the government through the recent visit of a joint committee of specialists on index numbers consisting of Professors Allyn A. Young, Wesley C. Mitchell, Irving Fisher and Edmund E. Day, who have gone over thoroughly the index numbers of these departments and have presented definite recommendations for their improvement and standardization in accordance with the most approved practices.

INCREASED RECOGNITION OF STATISTICS

The effort to make the *Survey of Current Business* a more complete picture of business conditions has resulted in almost tripling the number of items carried, even when some of the less important items, which formerly could be given separately, had to be combined, while the section on the import and export trade of foreign countries had to be eliminated altogether, with the exception of Canada and Great Britain, as the figures were generally too late to be of much use in a survey of "current business," and a great deal of space was used in their presentation. About 1,400 separate movements are now shown each month in the *Survey*.

The work of investigation that had to be done to secure these additional statistics bore fruit in a surprising manner. Contact with various government departments uncovered series of figures which had lain dormant for years, known to hardly any one but the clerk compiling them. Some of these figures proved highly useful to readers of the *Survey of Current Business*, and, as soon as an outside interest was shown, the departments realized the value of their heretofore unknown statistics and themselves issued special statements on the subject. A new angle on statistics was thus presented in many cases and the gradual permeation of this idea through the various bureaus, divisions and sections will make for better and more serviceable statistical data throughout the government service.

Trade associations and individual firms have also become more interested in the value of statistical work in their own industries after perceiving the data which other industries supply; they have set about to collect much figures for themselves or asked the Bureau of the Census to undertake the collection of current figures. Such requests are being received constantly and the Census Bureau is glad to put its facilities for collecting current data at the disposal of the industry provided the greater part of the individual firms will agree to report their facts regularly and promptly.

A still further recognition of the usefulness of statistical data through the publicity given in the *Survey of Current Business* is shown in the internal revenue figures on tax collections. As the taxes collected each month are percentages of the sales of the articles taxed, it has been generally a simple matter to compute the actual sales and thus show, for example, the wholesale value of candy sold by manufacturers each month or the money spent on theatre admissions. These have proved valuable figures to students of economic trends, and candy manufacturers have discovered a valuable index of their business. Unfortunately, Congress in revising the revenue act last spring gave no thought about the statistical value of these figures, and the candy data will hereafter be lost through the elimination of the tax on the manufacturers, while the theatre admission figures will not be comparable with previous data through revoking the tax as far as theatres charging less than 50 cents are concerned. Candy manufacturers, impressed with the value of the data, are, however, making an effort to have these figures collected through other sources in order to continue a comparable series.

The correspondence of the Bureau of the Census also shows an increased comprehension on the part of the general public of the use to

which these figures can be put, and specific instances of savings through scanning the data presented are numerous. It may not be too much to hope that the not far distant future may see fulfilled the vision of the Committee on Unemployment and Business Cycles of the President's Conference on Unemployment in its report published a year ago that the cyclical variations of business might be controlled through the increased volume and greater use of statistical data on various industries. The present statistical series, complete as they may appear in many respects, have only scratched the surface for the real foundation. More industries need to be covered, more data are needed on the industries already covered, the representative character of the figures must in many cases be increased and stricter definitions must be obtained for some existing data to clarify them and prevent duplication or confusion in the figures. All these matters must move slowly, for the process of education is long drawn out, but the Department of Commerce and other government agencies are bending their energies in this direction and slowly but surely the facts are beginning to come forth and be disseminated for the benefit of the business public.

A SHORTENED INTERPOLATION FORMULA FOR CERTAIN TYPES OF DATA

By E. P. Neale and D. M. Y. Sommerville, *Victoria University College, Wellington, New Zealand*

Where y is a continuous function of x, and we have four pairs of values: (x_0, y_0); (x_1, y_1); (x_2, y_2); (x_3, y_3); and it is sought to find the value of y (y_m) which corresponds to the value x_m of x, there exist various methods of interpolation. In the absence of knowledge of the law connecting the two series, it is usual to assume that the relation $y=a+bx+cx^2+dx^3+\ldots$ holds. Corresponding pairs of values of x and y can be successively substituted in this equation, which is then laboriously solved for a, b, c, and d; y_m can then be arrived at by substitution of the values obtained for a, b, c, and d, and the given value of x_m. Various modifications of this method exist, possibly the best known being the employment of Lagrange's Formula. All of these methods involve considerable calculation.

Our experience has shown that in a large proportion of interpolations the practical statistician is called upon to perform, the relation $x_m=\frac{1}{2}(x_0+x_3)=\frac{1}{2}(x_1+x_2)$ holds. This leads to a search for a simplified interpolation formula applicable to such cases.

By hypothesis, we have:

$$x_m=\tfrac{1}{2}(x_0+x_3) \quad \ldots\ldots\ldots \quad (1)$$
$$x_m=\tfrac{1}{2}(x_1+x_2) \quad \ldots\ldots\ldots \quad (2)$$

Let $z_0=\tfrac{1}{2}(x_0-x_3)$ ..... (3)

$$z_1=\tfrac{1}{2}(x_1-x_2) \quad \ldots\ldots\ldots \quad (4)$$

Adding (1) and (3), we have $x_0=x_m+z_0$ ..... (5)

Adding (2) and (4), we have $x_1=x_m+z_1$ ..... (6)

Subtracting (4) from (2), we have $x_2=x_m-z_1$ ..... (7)

Subtracting (3) from (1), we have $x_3=x_m-z_0$ ..... (8)

Moreover,

$$y_0=a+bx_0+cx_0^2+dx_0^3 \quad \ldots\ldots\ldots \quad (9)$$
$$y_1=a+bx_1+cx_1^2+dx_1^3 \quad \ldots\ldots\ldots \quad (10)$$
$$y_2=a+bx_2+cx_2^2+dx_2^3 \quad \ldots\ldots\ldots \quad (11)$$
$$y_3=a+bx_3+cx_3^2+dx_3^3 \quad \ldots\ldots\ldots \quad (12)$$

Adding (9) and (12), we have

$$y_0+y_3=2a+b(x_0+x_3)+c(x_0^2+x_3^2)+d(x_0^3+x_3^3)$$

Substituting (5) and (8), we have

$$y_0+y_3=2a+2bx_m+2c(x_m^2+z_0^2)+2d(x_m^3+3x_mz_0^2)$$
$$=2y_m+2z_0^2(c+3dx_m) \quad \ldots\ldots\ldots \quad (13)$$

Similarly, by adding (10) and (11), and substituting (6) and (4), we have

$$y_1+y_2=2y_m+2z_1^2(c+3dx_m) \quad \ldots\ldots\ldots \quad (14)$$

Multiplying (13) and (14) by z_1^2 and $-z_0^2$ respectively, and adding, we have

$$z_1^2(y_0+y_3)-z_0^2(y_1+y_2)=2y_m(z_1^2-z_0^2).$$

Therefore,

$$y_m=\frac{z_1^2(y_0+y_3)-z_0^2(y_1+y_2)}{2(z_1^2-z_0^2)};$$

which is our required simplified formula, applicable in all cases where the conditions set forth in (1) and (2) above hold good.

This formula is merely a special case ($n=1$) of the generalized result where the conditions

$$x_m=\tfrac{1}{2}(x_0+x_{2n+1})=\tfrac{1}{2}(x_1+x_{2n})= \quad . \quad . \quad . \quad =\tfrac{1}{2}(x_n+x_{n+1})$$

hold. This generalized result is

$$y_m=\begin{vmatrix} Y_0 & u_0 & u_0^2 & \ldots. & u_0^n \\ Y_1 & u_1 & u_1^2 & \ldots. & u_1^n \\ \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots \\ Y_n & u_n & u_n^2 & \ldots. & u_n^n \end{vmatrix} \div \begin{vmatrix} 1 & u_0 & u_0^2 & \ldots. & u_0^n \\ 1 & u_1 & u_1^2 & \ldots. & u_1^n \\ \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots \\ 1 & u_n & u_n^2 & \ldots. & u_n^n \end{vmatrix}$$

where $Y_0=\frac{1}{2}(y_0+y_{2n+1}), \ldots . \quad Y_r=\frac{1}{2}(y_r+y_{2n+1-r});$
$u_0=(x_0-x_{2n+1})^2, \ldots . \quad u_r=(x_r-x_{2n+1-r})^2;$

n and t being positive integers.

Where there are six pairs of values of x and y, $n=2$; and

$$y_m=\frac{Y_0(u_1u_2^2-u_1^2u_2)+Y_1(u_2u_0^2-u_0u_2^2)+Y_2(u_0u_1^2-u_0^2u_1)}{u_1u_2(u_2-u_1)+u_2u_0(u_0-u_2^2)+u_0u_1(u_1-u_0)};$$

where $y_0=\frac{1}{2}(y_0+y_5); Y_1=\frac{1}{2}(y_1+y_4); Y_2=\frac{1}{2}(y_2+y_3);$
$u_0=(x_0-x_3)^2;\ u_1=(x_1-x_4)^2;$ and $u_2=(x_2-x_3)^2.$

Apply these formulae to the import data for the earlier years of the second decade of the last century, quoted by Dr. A. L. Bowley (*Elements of Statistics*, Fourth Edition), page 234:

Year	*Value of imports in thousands of pounds*
1810	39202
1811	26510
1812	26163
1813	(Records destroyed by fire)
1814	33755
1815	32987
1816	27431

Using the years 1811 to 1816 only, we have $z_1=1$; $z_0=2$; $y_0=26510$; $y_1=26163$; $y_2=33755$; $y_3=32987$. By the formula, $y_m=30029$.

Using the years 1810 to 1816, we have, measuring the values of x in biennia (not years), $Y_0=33316\frac{1}{2}$; $Y_1=29748\frac{1}{2}$; $Y_2=29959$; $u_0=9$; $u_1=4$; $u_2=1$. By the formula, $y_m=30421$. The saving in time and ease of calculation is considerable.

WAGE INDEXES AND PRICE INDEXES

By George Soule

In the June number of the Journal, in the interesting paper by Carl Snyder on "A New Index of the General Price Level From 1875," appears an assumption common to many discussions of the price level, which seems open to question. This assumption is that an index of wages is an index of the price of labor, coördinate as a measure of fluctuations in the general price level with indices of wholesale prices, cost of living, etc.

Virtually all indices of wages are based on data showing the amounts paid to wage-earners per hour or per week. But when an employer

buys an hour's or a week's labor, he is not buying a standardized and comparatively uniform article like a ton of steel or a bushel of potatoes. The value of the time-unit of labor may vary, not only with fluctuations of the general price level, but with changes in the productive efficiency of the labor. In other words, the unit is an expanding or a contracting one, and has about as much usefulness in a price index as would a ton which varied from 1,500 pounds in one year to 2,500 pounds ten years later. If we want to gauge accurately the price factor in wages, it will be necessary to derive an index from *unit labor costs* in the production of standardized articles. With such an index it would be possible, for instance, to compare the course of the price of the labor in a ton of steel with the course of the price of a ton of steel or with the course of pig-iron prices, or with any commodity price index.

Most indices of wages and wage-earnings which we have are virtually indices of personal incomes, rather than indices of prices. Mr. Snyder will concede, I think, that it would be misleading to use indices of the salaries of business executives, or of the individual returns from investments, or of the national income per capita, as indices of the general price level. Such indices doubtless are affected by changes in the price level, but they also are affected by changes in productivity, or in the total national real income, and in the distribution of that income. The same qualifications apply to indices of wage incomes.

The theoretical importance of the distinction is large. Aside from temporary fluctuations in the efficiency of labor, there is a decided long-term tendency towards an increase in such efficiency. An hour of labor in the steel industry, for instance, produces far more in 1924 than it did in 1875. The same tendency exists in most other industries. As a result the national per capita product tends to rise. This brings about, as a necessary consequence, not only an increase in goods produced per capita, but an increase in per capita purchasing power — otherwise there would not be the effective demand to draw forth the production of the goods. But if purchasing power increases, incomes increase more rapidly than prices. And if wage-earners are to share in the general increase in purchasing power, their wages must likewise increase, in the long run, more than prices. Unless we are to assume that wage-earners, alone among the various elements in the population, cannot share in the benefits of improved technique, we must assume that there may be a secular trend in wages, having some relation to productivity, which does not exist in the course of the general price level. If there were no changes in the sharing of the national income as among wage-earners and other classes of the community, this trend would be the same as the trend of national per-capita production. In

so far as there are changes in sharing, the trend differs from the trend of productivity.

The unfortunate practical consequence of the assumption that wage trends vary with price trends is that it leads to the assumption that they ought to do so, and hence that wages should be "deflated" as much as prices when prices fall, or should not rise more than prices when prices rise, and hence that labor should not share in the general growth of productivity, wealth and purchasing power.

An examination of Mr. Snyder's chart on page 192 reveals that although the wage curve was some distance below the wholesale price curve in 1875, it was above it by an almost equal amount in 1923. That suggests that labor has to some extent shared in the increase of productivity from the beginning to the end of the period. But the intervening years reveal that the relationship of wages and wholesale prices went through several phases. Between 1875 and 1897 the price trend was on the whole downward, while the wage trend was slightly upward. Between 1898 and 1907 prices rose more rapidly than wages. From 1908 to 1913 they were about parallel. From 1914 to 1920 wages lagged far behind. And wages registered a marked gain after 1920. Therefore, either the curve of productivity or the distribution of income was so irregular that it would be extremely difficult to eliminate any secular trend of wages due to productivity or distribution so that the remainder might indicate the general price level. On the whole, it seems best not to compare existing indices of wages with indices of prices, except to discover changes in the purchasing power of wage-earners.

A SEX CLEAVAGE IN THE PRESIDENTIAL ELECTION OF 1920

A question of standing interest to students of politics and government is the use made by women of the ballot. The writers have elsewhere summarized evidence to show that women, as compared with men, tend to support the "moral" and "good government" side of public issues when they cast their ballots, and that they likewise tend to oppose more strongly the "radical" side.[1]

The evidence upon which this conclusion is based is for the most part obtained from local or state elections, where "moral" and "radical" issues are more likely to be presented to the voters in such a form that clear choices are possible. With respect to the attitude of women toward the presidential candidates in a national election, there is but one bit of evidence available. Yet in some respects this single piece of evidence is so striking that the writers feel it deserves the attention of those who are interested in the exact measurement of political phenomena.

[1] "American Women's Ineffective Use of the Ballot," *Current History Magazine*, July, 1924.

buys an hour's or a week's labor, he is not buying a standardized and comparatively uniform article like a ton of steel or a bushel of potatoes. The value of the time-unit of labor may vary, not only with fluctuations of the general price level, but with changes in the productive efficiency of the labor. In other words, the unit is an expanding or a contracting one, and has about as much usefulness in a price index as would a ton which varied from 1,500 pounds in one year to 2,500 pounds ten years later. If we want to gauge accurately the price factor in wages, it will be necessary to derive an index from *unit labor costs* in the production of standardized articles. With such an index it would be possible, for instance, to compare the course of the price of the labor in a ton of steel with the course of the price of a ton of steel or with the course of pig-iron prices, or with any commodity price index.

Most indices of wages and wage-earnings which we have are virtually indices of personal incomes, rather than indices of prices. Mr. Snyder will concede, I think, that it would be misleading to use indices of the salaries of business executives, or of the individual returns from investments, or of the national income per capita, as indices of the general price level. Such indices doubtless are affected by changes in the price level, but they also are affected by changes in productivity, or in the total national real income, and in the distribution of that income. The same qualifications apply to indices of wage incomes.

The theoretical importance of the distinction is large. Aside from temporary fluctuations in the efficiency of labor, there is a decided long-term tendency towards an increase in such efficiency. An hour of labor in the steel industry, for instance, produces far more in 1924 than it did in 1875. The same tendency exists in most other industries. As a result the national per capita product tends to rise. This brings about, as a necessary consequence, not only an increase in goods produced per capita, but an increase in per capita purchasing power — otherwise there would not be the effective demand to draw forth the production of the goods. But if purchasing power increases, incomes increase more rapidly than prices. And if wage-earners are to share in the general increase in purchasing power, their wages must likewise increase, in the long run, more than prices. Unless we are to assume that wage-earners, alone among the various elements in the population, cannot share in the benefits of improved technique, we must assume that there may be a secular trend in wages, having some relation to productivity, which does not exist in the course of the general price level. If there were no changes in the sharing of the national income as among wage-earners and other classes of the community, this trend would be the same as the trend of national per-capita production. In

so far as there are changes in sharing, the trend differs from the trend of productivity.

The unfortunate practical consequence of the assumption that wage trends vary with price trends is that it leads to the assumption that they ought to do so, and hence that wages should be "deflated" as much as prices when prices fall, or should not rise more than prices when prices rise, and hence that labor should not share in the general growth of productivity, wealth and purchasing power.

An examination of Mr. Snyder's chart on page 192 reveals that although the wage curve was some distance below the wholesale price curve in 1875, it was above it by an almost equal amount in 1923. That suggests that labor has to some extent shared in the increase of productivity from the beginning to the end of the period. But the intervening years reveal that the relationship of wages and wholesale prices went through several phases. Between 1875 and 1897 the price trend was on the whole downward, while the wage trend was slightly upward. Between 1898 and 1907 prices rose more rapidly than wages. From 1908 to 1913 they were about parallel. From 1914 to 1920 wages lagged far behind. And wages registered a marked gain after 1920. Therefore, either the curve of productivity or the distribution of income was so irregular that it would be extremely difficult to eliminate any secular trend of wages due to productivity or distribution so that the remainder might indicate the general price level. On the whole, it seems best not to compare existing indices of wages with indices of prices, except to discover changes in the purchasing power of wage-earners.

A SEX CLEAVAGE IN THE PRESIDENTIAL ELECTION OF 1920

A question of standing interest to students of politics and government is the use made by women of the ballot. The writers have elsewhere summarized evidence to show that women, as compared with men, tend to support the "moral" and "good government" side of public issues when they cast their ballots, and that they likewise tend to oppose more strongly the "radical" side.[1]

The evidence upon which this conclusion is based is for the most part obtained from local or state elections, where "moral" and "radical" issues are more likely to be presented to the voters in such a form that clear choices are possible. With respect to the attitude of women toward the presidential candidates in a national election, there is but one bit of evidence available. Yet in some respects this single piece of evidence is so striking that the writers feel it deserves the attention of those who are interested in the exact measurement of political phenomena.

[1] "American Women's Ineffective Use of the Ballot," *Current History Magazine*, July, 1924.

The ballots of men and women are separately recorded, so far as the writers are aware, in the state of Illinois alone. In the presidential election of 1920 Illinois was carried by the Republican party with an overwhelming majority. Of the men's votes for Harding and Cox combined, Harding received 71.4 per cent. Of the women voting for the two candidates, 74.6 per cent cast their ballots for Harding.

Now the outstanding thing about the support given the Republican candidate by the women is not its proportionately greater extent in the state at large but the constancy with which the percentage of the Harding vote among the women exceeded the percentage of the Harding vote among the men in all parts of the state. The percentage of the combined Harding-Cox vote received by Harding was calculated in the case of each of the 102 counties in Illinois, and the city of Chicago. The percentages received by Harding varied from less than 50 per cent to more than 90 per cent. Yet the corresponding percentages of the women's votes and the men's votes, when correlated produced the remarkably high coefficient

$$r = .964 \pm .06$$

Class intervals of 5 per cent were employed in the calculation. Thus in every county in the state, with two exceptions (Union and Green counties), the proportion of Harding votes among the women represented a slight but constant excess over the proportion of Harding votes among the men.

In the absence of conditions existing in every part of the state of Illinois and peculiar to it alone which might account for the high value of r—and no such conditions are known—it seems to the writers legitimate to conclude that there was a general tendency toward sex cleavage in the 1920 election. In other words, we believe that women in general, at least in the northern states, were more strongly Republican than the men. The contrary assumption, that no such cleavage existed in states other than Illinois, necessitates the discovery of a factor or factors operating in all parts of that state and in that state alone to throw more women's votes to Harding. The well attested facts that women are on the "moral" and conservative sides in local elections, combined with the circumstance that Harding of the two candidates was widely believed to be the more conservative and the more "dry," would together give this statistical conclusion additional weight.

MALCOLM M. WILLEY AND STUART A. RICE

Dartmouth College

THE FIFTIETH ANNIVERSARY OF QUETELET'S DEATH

Fifty years have passed since the death of one of the most remarkable men to whom Belgium has given birth: Quetelet. It is very much to be regretted that this important anniversary should have been forgotten officially. Yet we have here a pure and universally appreciated scientific reputation. Besides, it would have been an opportunity to recall the example of a life devoted to science and animated by a glowing and strengthening enthusiasm.

At seventeen, Quetelet began teaching at a private college in Audenairde, a small town in Flanders. Six years later he took the degree of doctor of science at the University of Ghent with a remarkable dissertation in the field of geometry. The following year he was received as a member of the Royal Academy. In 1823, he suggested that the Secretary of Public Instruction should create an observatory in the southern provinces, actual Belgium forming at that time a part of the Netherlands. After much patience, perseverance and energetic effort, this project was realized a few years later. Quetelet was appointed as head of the new institution and given complete charge of its organization.

But the attention and appreciation of the learned world had been directed toward him more particularly by his statistical work. His first paper on demography, "*Sur les lois des naissances et de la mortalité à Bruxelles*," was published in 1825. Ten years later his fundamental work, "*Essai de physique social* (*sur l'homme et le développement de ses facultés*)," was published in Paris. It is a synthesis of Quetelet's statistical work. In that book the author examines the average man in his physical and moral aspects. This audacious theory was taken up again and completed in another work, "*Du système social et les lois qui le régissent*" (1848). In 1841, Quetelet was called to direct the activities of the Central Commission of Statistics, which under his guidance developed into a very famous institution. The last years of his life were devoted to an active and fruitful participation in the work of the international meetings for statistics and to the preparation of the second edition of his *Social Physics* (1869). He died on the 17th of February, 1874.

Without being transcendental, Quetelet's mathematical work is interesting and original. His researches concerning the theory of caustics opened the way for important investigation. As a result of his efforts the first Belgian periodical devoted to exact science was founded—*La correspondance mathématique et physique*. Finally he is the author of a documentary work worthy of notice, "*Histoire des sciences et mathématiques chez les Belges*." Quetelet's activities in the field of astronomy consisted in fruitful investigation along several lines, such as descriptive astronomy, magnetism, meteorology, terrestrial physics, etc. It is necessary to mention specially his great work on the climate of Belgium. But his universal renown rests, without doubt, upon his statistical work. His name is closely linked with the history and development of the science of statistics. His numerous papers on demography and his remarkable analytical and synthetic studies of statistics are important works in many ways. He still remains an authority on statistical method. He was also its apostle. The work which he accomplished in statistics is fundamental and considerable. Therein lies his chief scientific merit. In a general way, Quetelet's statistical doctrine can be thus characterized: for a monographical description of facts, for a systematic census of elements, he substituted a scientific observation of the masses, a methodical survey of groups, founded on the principles of the theory of probabilities. The fundamental idea which guided all his statistical investigation was that statistical method is the application of the calculus of probabilities to the observation of facts. Sixty years ago this was an audacious and original doctrine. It seems that Quetelet was right. His paper of 1845, "*Sur l'appréciation*

des documents statistique et la théorie des moyennes," was the first step in a science which today is called biometry. On the other hand, the prodigious rise and important development of mathematical statistics justify Quetelet's contention that the statistical method lay in the application of the calculus of probabilities.

For Quetelet, statistics was a science. This being so, it was not sufficient to collect documents and numbers, taking care only to group them so as to facilitate comparisons. That was administrative statistics. There was also the scientific side which seeks to evaluate these documents and to draw conclusions from them. A hundred years ago Fourier said: "Statistical research will make no real progress until they are performed by those who have thoroughly investigated mathematical theories." It has been rightly said that Quetelet's *average man* died before his author. The *average man*, far from being the type of the species, is an impossibility. Cournot and Bertillon have given a justifiable and definite criticism of this theory of "type." And the theory of the *average moral man*, a pure mathematical fiction, is today quite forgotten.

But Quetelet, endowed with a superior mind, a high and clear intelligence, being a real scientist, an obstinate investigator, has given a vital impulsion to the study of science in Belgium. His work is complex; it abounds in precious and deep teachings. Quetelet was a real statistician. For more than a quarter of a century he was a leading figure in the intellectual life of Belgium.

CONSTANT LURGUIN

Fellow at the University of Brussels

WELFARE ACTIVITIES OF THE UNITED STATES STEEL CORPORATION

The Bureau of Safety, Sanitation and Welfare of the United States Steel Corporation has issued a brief report, under date of July, 1924, which contains some extremely interesting statistics revealing the extent and nature of modern corporate welfare activities.

Since January 1, 1912, to the end of 1923 the company has expended $126,885,000 on account of welfare activities, of which the major amounts have been paid out in relief for injured men and the families of men killed, reaching a total of $37,000,000. For sanitation the company expended about $25,000,000 and for playgrounds, schools, gardens, visiting nurses, etc., $22,000,000 more. The amount expended on account of accident prevention reached nearly $12,000,000, while $18,500,000 represents the company's contribution to the employees' stock subscription plan. For other activities, chiefly contributions to pension funds, $12,000,000 more was expended.

The aggregate and total, in its social and economic results to the welfare of the workers concerned represents, however, a vastly greater contribution to human well-being, for it is shown by a record, dating back to 1906, which is assumed as a base, that since that year 39,374 employees have been saved from serious injury. Contrasting the record of 1906 with 1923 the accident experience of that year was 55.4 per cent less. Disabling accidents, regarding which

records have been kept since 1912, show a decrease during 1923 of 70 per cent. During the entire period, 244,588 persons were saved from injuries, ranging from miner impairments to total disability and death.

According to a general classification of 284,494 accidents, hand labor caused 43.7 per cent of the total, followed by mines with 15.7 per cent, falls 8.3 per cent, burns 6.9 per cent, while eye injuries represent 5.2 per cent, machinery injuries 4.9 per cent and miscellaneous causes 10.9 per cent. It would require a more extended analysis of this vast experience to trace the accident factor in detail and the effects of various safety-first measures as affecting the trend of the accident rate. The large proportion of accidents caused by hand labor suggests the great importance of this element in measures aiming at accident prevention. Of the accidents caused by hand labor 14.5 were from falling materials, followed by accidents due to sledge hammers represented by 7.1 per cent and accidents due to being caught between materials represented by 6.0 per cent. In no direction perhaps have modern safety-first efforts been more effective than in reducing the accident liability to hand labor. Of the machinery accidents 3.5 per cent were caused by the operator being caught or struck by machinery, while 1.2 per cent represent the accidents due to the operator being struck by materials. The percentages for hand and machinery labor are, of course, the proportion of the total of the 284,494 accidents, classified as to cause.

According to a brief summary of the facilities constructed or installed for the use of employees, as of December 31, 1923, the following items are of exceptional interest: The number of dwellings and apartment houses constructed and leased to employees at low rental rates was 28,451. The Corporation maintains 25 churches, 33 schools, 40 clubs and 66 restaurants, aside from 283 rest and waiting rooms. It also maintains 175 playgrounds, 19 swimming pools, 125 athletic fields, 112 tennis courts and 21 band stands. In addition it maintains 29 practical housekeeping centers.

The Corporation has 13 base hospitals and 389 emergency stations, aside from 66 training stations (first aid and rescue services). Its medical staff includes 234 surgeons, physicians, and internes, 110 outside surgeons on salaries, 235 nurses, 112 orderlies, and 71 visiting nurses. In addition there are 44 sanitary inspectors, 130 safety inspectors, spending their entire time on the work, and 84,491 employees who have served on safety committees. The number of employees trained in first aid and rescue work is 20,719, while the number of employees now in training for this purpose is given as 863.

The bulletin contains a large number of exceedingly interesting illustrations vizualizing the statistical facts, which would otherwise fail of being thoroughly understood.

Of special interest are the illustrations of employees' hospitals, which form an inspiring example of modern hospital development. It would seem difficult to improve what is being done in this direction. Other interesting illustrations have reference to the water supply of different Steel communities, which in every case are under technical supervision.

In an age of widespread unrest and much corporate antagonism, the welfare activities of the United States Steel Corporation, all of which are on a purely

voluntary basis, representative of the Corporation's good will towards its employees, are an illuminating example of the trend of things which may be looked upon as the most successful means of meeting socialistic and communistic objections to the existing order.

The Bureau of Safety, Sanitation and Welfare is in charge of Mr. Charles L. Close, Manager, to whom largely belongs the credit for the success of the results achieved; but the welfare policy of the company represents primarily the broad vision of Judge Elbert H. Gary, the Chairman of the United States Steel Corporation, who for many years has been active in other lines of welfare and safety to the immeasurable benefit of the American people.

F. L. HOFFMAN

Wellesley Hills, Massachusetts

PROGRESS OF WORK IN THE CENSUS BUREAU

CENSUS OF AGRICULTURE

The plans for the census of agriculture, now being perfected, contemplate the appointment of about 200 supervisors and 17,500 enumerators. The coöperative arrangements with the Department of Agriculture cover the field work as well as the formulation of the schedule, with the result that 35 of the supervisors will be state statisticians detailed for this work by that department; that the forest rangers will enumerate most farms on forest reserves and on many of the adjoining areas; and that the reclamation service will enumerate the farms in those irrigation districts which are under government control. Of the other supervisors 78 will be detailed from the regular employees of the Bureau, and the remainder, about 90, will be specially appointed, most of them upon the recommendation of the state agricultural colleges. The enumerators in general will be appointed by the Director of the Census upon the recommendation of the supervisor. They will be paid on a piece-price basis, or in other words so much per schedule, the amount being usually 25 or 30 cents. In many cases the enumeration in the northern latitudes and in mountainous regions will be begun in November, so as to anticipate the deep snows which may come later. Elsewhere the canvass will begin about December 1 and will take probably about two months.

CENSUS OF INSTITUTIONAL CLASSES

The machine tabulation of the data relating to the institutional classes, including convicted prisoners, insane in hospitals, feeble-minded and epileptic in institutions, paupers in almshouses, orphans or other children under institutional care, and hospitals and dispensaries, has been completed, and the report on hospitals and dispensaries and that on paupers have been sent forward to the printer. The report on children under institutional care is being prepared by Georgia G. Ralph, formerly of the Russell Sage Foundation, that on juvenile delinquents by Katharine F. Lenroot of the Children's Bureau, that on insane by Horatio M. Pollock of the New York Hospital Commission, and that on the feeble-minded and epileptics by Edith M. Furbush of the National Committee for Mental Hygiene.

CENSUS OF WEALTH, DEBT, AND TAXATION

Copy for all the reports on the wealth, debt, and taxation census has been sent to the Government Printing Office, and the Digest of State Laws relating to Taxation and Revenue has been printed. The bureau some time since issued four printed leaflets giving state totals relating respectively to estimated wealth, public indebtedness, taxes, and assessed valuation.

J. A. H.

MISCELLANEOUS NOTES

BUREAU OF LABOR STATISTICS NOTES

Automobile Production.—Some interesting conclusions on production in the automobile industry are set forth in an article by Mr. Mortimer W. La Fever on "Workers, Machinery, and Production in the Automobile Industry," which appeared in the October issue of the *Monthly Labor Review*.

The changes in methods of manufacturing in automobile plants have been so recent and so rapid that it was possible in several establishments to secure accurate records of exactly what has been done. For instance, in one establishment the records show that whereas it required 4,664 one man hours to produce a car in 1912 the same establishment in 1923 was putting out a car with all the improvements that had been made upon it in the meantime in 813 one man hours. Another establishment that was putting out a car for each 1,260 one man hours in 1912 was in 1923 producing a car for each 228 one man hours. Mr. La Fever goes into very great detail in this paper to show at what particular points in the manufacture these time savings have been made and by what methods.

Cost of Shoe Manufacturing.—With the publication of its Bulletin No. 360 the United States Bureau of Labor Statistics announces the result of its study of "Time and Labor Costs in Manufacturing 100 Pairs of Shoes, in 1923." This is the result of a somewhat extensive investigation made during the summer of 1923, the principal field work as well as office compilations having been done by Mr. P. L. Jones of the bureau. Representative shoe manufacturing establishments located in nine important shoe producing States were covered in the investigation. The material is given in great detail, showing time and labor cost by occupations. Ignoring occupational costs the total averages would indicate that to make a single pair of men's Goodyear welt high shoes in New England requires 79 minutes one man's time and costs $0.84. In the central Atlantic States it costs 61 minutes one man's time and $0.57. In the middle Western States it costs 89 minutes one man's time and $0.92. The time cost in New England for one pair of women's Goodyear welt low shoes is 77½ minutes one man's time and the labor cost is $0.78. In the central Atlantic States the time cost is two hours and the labor cost is $1.04; while in the middle West the time cost is 87 minutes and the labor cost $0.88. For a woman's McKay low shoe in New England the time cost is 70 minutes and the labor cost $0.69. In the central Atlantic States it is 77 minutes time cost and $0.69 labor cost. In the middle West the time cost is 94 minutes and the labor cost $0.92. The Bureau of Labor Statistics has already published similar cost figures for the year 1916, and it is interesting to compare the figures for an identical establishment, which show that between 1916 and 1923 there was a decrease in time cost of 25.1 per cent, while there was an increase of 48.5 per cent in labor cost; and while wages had increased considerably more than 48.5 per cent the shock of wage increase had been lessened by a 25 per cent increase in productivity.

Efficiency in Bricklaying.—In the November *Monthly Labor Review* Commissioner Ethelbert Stewart presents the results of the investigation made by the United States Bureau of Labor Statistics into the efficiency of labor in the building trades. In bricklaying, five southern cities, three eastern cities and seven western cities, (counting Minneapolis and St. Paul as one) were covered. The labor efficiency in bricklaying, plastering, painting and roofing are discussed. The investigation was orginally intended to include carpentering, but that was abandoned because of the difficulty in determining the definite units. In bricklaying the unit is 1,000 common brick laid in a straightaway dead wall. Commissioner Stewart does not find that the question of union or non-union or mixed groups of workers materially affects the output per man per hour within a given locality. It is interesting to note in view of the quite general newspaper comments on the loafing habits of the workers in the building trades, particularly bricklayers, that Commissioner Stewart states that in no case in any part of the country did the bureau agents find a single contractor who had a record or who had ever kept a record of the actual number of bricks placed in the wall per one man hour of bricklayer's time, and that the field agents of the bureau were obliged to arrange with contractors, superintendents and foremen to keep such time for the especial use of the bureau in these studies. A text table which groups the information secured in bricklaying will give an idea of the general scope of the article.

BRICKLAYING (AVERAGE BY CITIES)

City	Number of schedules	Rate of pay per hour	Number of brick laid per hour	Number of brick laid per day of 8 hours	Cost of laying 1,000 brick
Atlanta, Ga.	23	$1.05	185.3	1,482.4	$6.30
Birmingham, Ala.	8	1.16	241.0	1,928.0	4.82
Chattanooga, Tenn.	11	1.46	226.1	1,808.8	6.85
New Orleans, La.	7	1.01	203.5	1,628.0	5.11
Norfolk, Va.	10	1.37½	231.8	1,854.4	6.38
Boston, Mass.	5	1.25	97.7	781.6	12.94
New York, N. Y.	12	1.76	157.6	1,260.8	11.69
Philadelphia, Pa.	14	1.53	123.3	986.4	12.65
Chicago, Ill.	24	1.57	156.8	1,254.4	10.60
Cincinnati, Ohio	10	1.43	131.5	1,052.0	11.47
Cleveland, Ohio	14	1.50	147.8	1,182.4	10.55
Denver, Colo.	18	1.50	212.4	1,699.2	7.58
Detroit, Mich.	15	1.54½	154.0	1,232.0	10.44
Indianapolis, Ind.	10	1.36½	95.7	765.6	14.47
Minneapolis and St. Paul, Minn.	21	1.21 1/10	193.2	1,545.6	6.84

Cost of Living Studies.—In Bulletin No. 357, entitled, "Cost of Living in the United States," the United States Bureau of Labor Statistics has finally published in as much detail as was found to be possible the results of the family budget survey made in 1918–19 of 12,096 white families in 92 localities in the 42 States covered. Special studies covering certain phases of this investigation have been published from time to time until the material is more or less familiar to all. It was hoped that the bureau would be able to publish these schedules in complete detail as was done in the case of a former family budgetary study, but the cost of printing made this prohibitive. The titles of the tables indicate what has in fact been done:

Table A.—Sources and amounts of family incomes in one year, by cities and income groups.

Table B.—Average quantity of specified articles of food consumed per family in one year, by geographical divisions.

Table C.—Quantity of and expenditure for clothing purchased per family in one year, by sex, age, and income groups.

Table D.—Number of families living in houses or apartments having specified number of rooms, and average amount paid for rent in one year, by cities and income groups.

Table E.—Quantity of and expenditure for fuel and light per family in one year, by cities, and income groups.

Table F.—Quantity of and expenditure for furniture and house furnishings per family in one year, by income groups.

Table G.— Expenditure for miscellaneous items in one year, by geographical divisions and income groups.

It will thus be seen that the family income has been made substantially the unit of tabulation and this is given by cities and geographical divisions. Bulletin No. 357 is the final disposition of the family budget material as collected by the Bureau of Labor Statistics in 1918–19.

Up to the present time there have been no Governmental reports on wages and hours of labor in multifarious mining, and the United States Bureau of Labor Statistics has just added this industry to its list for studies of wages. The mining of iron ore, copper, lead, zinc, and gold and silver lode mining will be covered in the study of this industry.

New Secretary of the Association Appointed.—At the last annual meeting, the nominating committee was unable to present a candidate for the office of secretary-treasurer. The Association authorized the Board of Directors to appoint a secretary-treasurer as soon as the proper person could be found. Under this authorization the board, at a meeting held October 11, 1924, appointed Willford I. King, of the National Bureau of Economic Research, New York City, as secretary. The acting treasurer, Robert E. Chaddock, has agreed to continue in that office until the annual meeting in December, when the fiscal year closes. The board is happy to announce to the members that Dr. King has consented to assume his duties on November 1, 1924. The retiring secretary takes this opportunity to express his appreciation of the hearty coöperation of the membership, during his term of seven years, which has made so agreeable his relations with the Association.

ROBERT E. CHADDOCK
Acting Secretary

Professor Westergaard Retires.—After forty-one and a half years in the service of the University of Copenhagen, Professor Harald Westergaard has retired from his chair. His last lecture, which was held on May 28, dealt with the "Future of Statistics," a sketch of the present status of this science and an outline of the main problems which most urgently needed solution.

Professor Westergaard not only taught statistics but also taught economics, and although the former was his chief work, it was decided by the faculty to appoint a professor of political economy with a view of later finding a lecturer in theory of statistics. Under the given conditions this was probably the best way of promoting the regular courses in the faculty.

It may be of interest to American readers to know the procedure followed in the University of Copenhagen in appointing a professor as successor to Professor Wester-

gaard. Quite naturally this must be quite different in a small country with only three millions of inhabitants than in a large country like the United States where there would probably be many more applicants. The procedure is rather complicated. At first Scandinavian economists were invited to apply for the position. (A Danish student can easily follow a Norwegian or Swedish lecturer.) Five economists answered this invitation: Mr. Rögina, lecturer in the polytechnical school, Dr. Mackeprang, a Norwegian economist, Dr. Schönkeyder, and two Swedes, Dr. Lindahl, lecturer in the University of Lund and Dr. Ohlin, who had just been appointed lecturer in the University of Stockholm. The ages of these competitors ranged from 25 to 50. The first step was a report from the Faculty of Law and Economy. As no one of the applicants was considered "*facile princeps*," the next step was the appointment of a committee which was to arrange a competition. This committee consisted of seven members: the chairman, Professor Berlin, being dean of the faculty. The other members were three Danish professors, Birch, Nielson and Westergaard, one of the official censors of the University Examinations, Mr. Trap, and Professor Eli Hechsaker (Stockholm) and Professor Agrum (Christiania). It was the duty of this committee to arrange the competition and also to make itself acquainted with the published works of the various applicants. Mr. Rögina had written a large work on the alcohol legislation in Scandinavian countries, and a manual on the Danish system of taxes; Dr. Mackeprang had been chiefly interested in labor and the theory of statistics; Dr. Schönkeyder had written on capital and was the author also of many articles on rent and taxation. Dr.Lin dahl had written a book on "Die Gerechtigkeit des Besteuerung," and in the course of the summer he published a book on monetary politics; whereas Dr. Ohlin had just published a book on international trade.

The competitors were allowed three months to write an essay on the effect of a general reduction of hours of labor on production and supply of goods. Mr. Rögina having withdrawn, four printed investigations were submitted to the committee, to be published later. All of these essays dealt very ably with the question of the effect of reduction of hours of labor on the production in an individual plant while they entered somewhat less into the more difficult general problem. In the middle of September each competitor was required to give two public lectures, the subject of which he was allowed to choose himself. Dr. Lindahl spoke on capital and interest, dealing chiefly with the thcories of Bohn-Bawerk and Widersoll; Dr. Mackeprang on the history of mathematical economy, dealing with Cournot, Gassen, Jevons, Walres and Pareto; Dr. Ohlin lectured on the stabilization of the currency and Dr. Schönkeyder on exact methods of investigation, as applied especially to economics. Finally the competitors were allowed two days' preparation for a subject common to them all, *viz.*, guild socialism.

After these lectures a lengthy discussion took place in the committee. Two of the members voted for Dr. Lindahl, the others for Dr. Ohlin. The decision now rests with the Minister of Instruction, but will probably not differ from the vote of the majority of the committee.

The American Public Health Association.—The fifty-third annual meeting of the American Public Health Association was held on October 20–23, 1924, at Detroit, Michigan. Of particular interest to members of the American Statistical Association was the section of the program devoted to vital statistics. The following papers were read at this section:

FIRST SESSION

SOME STATISTICAL ASPECTS OF PNEUMONIA. Edwin Bidwell Wilson, *Professor of Vital Statistics, Harvard School of Public Health, Boston, Mass.*

WHAT THE PRACTICING PHYSICIAN FINDS OF VALUE IN VITAL STATISTICAL TABLES. Earl Ingram Carr, M.D., *Lansing, Mich.*

REPORT OF COMMITTEE ON PUBLIC HEALTH CLIMATOLOGY. *Chairman*, E. W. Kopf.
Discussion. C. C. Pierce, M.D., *Senior Surgeon, U. S. Public Health Service, Washington, D. C.*

SEASONAL VARIATION OF TUBERCULOSIS MORTALITY. Robert Watson, *National Tuberculosis Association, New York City.*

SECOND SESSION

MORBIDITY REPORTS WITH SPECIAL REFERENCE TO THE COLLECTION OF DATA. Bolivar J. Lloyd, M.D., *Assistant Surgeon General, U. S. Public Health Service, Washington, D. C.*
Discussion. S. J. Crumbine, M.D., *Director of Public Health Relations, American Child Health Association, New York City.*

TREND OF MORTALITY IN CHILDREN OF PRE-SCHOOL AGE, 1900–1922. E. W. Kopf, *Metropolitan Life Insurance Co., New York City.*

REPORT OF COMMITTEE ON CONTRIBUTORY CAUSES OF DEATH. *Chairman*, George H. Van Buren, *Metropolitan Life Insurance Company, New York City.*

REPORT OF COMMITTEE ON PROFESSIONAL STANDARDS FOR STATISTICIANS. *Chairman*, E. W. Kopf, *Metropolitan Life Insurance Co., New York City.*

REPORT OF COMMITTEE ON STANDARDS OF VITAL STATISTICS TEACHING. *Chairman*, John W. Trask, M.D., *U. S. Marine Hospital, Buffalo, N. Y.*

THIRD SESSION

(*Joint Session with Health Education and Publicity Section*)

THE PREPARATION OF VITAL STATISTICS FOR EDUCATIONAL USE. Donald B. Armstrong, M.D., *Metropolitan Life Insurance Company, New York City.*

THE USE OF VITAL STATISTICS IN HEALTH EDUCATION. Evart G. Routzahn, *Russell Sage Foundation, New York City.*
Discussion. Geo. T. Palmer, Dr.P.H., *American Child Health Association, New York City.*

Journal of Land and Public Utility Economics.—A new journal, the *Journal of Land and Public Utility Economics*, is to be issued under the editorship of Dr. Richard T. Ely, Professor of Economics, University of Wisconsin, and Director of the Institute for Research in Land Economics and Public Utilities. This journal will publish the results of the Institute's researches. It is also planned to draw contributing articles from authorities throughout the country. The magazine will be concerned with the economic problems of land utilization and public utility administration and regulation, as distinguished from the purely technical or engineering aspects of these subjects; and will contain eight or ten leading articles on such subjects as forecasting land values, the taxation of land, the significance of tax exemption to public utilities, a survey of farm mortgage interest rates throughout the country, real estate advertising, farm tenancy, cost of public utility financing. In addition to these leading articles, there will be three departments: namely, summaries of research and experiment, statements of pertinent legislation and court decisions, and book reviews. The journal will be published quarterly.

Brown Business Service.—A new series of reports on economic conditions has been recently begun under the title of *The Brown Business Service*, by the Brown Bureau of Business Research. This organization was begun three years ago, under the guidance

of Professor Henry B. Gardner, by Professor Ralph E. Badger of the Department of Economics in Brown University. Members of the Providence Chamber of Commerce coöperated with the University in forming the bureau.

After long consideration the Board of Directors decided that the intensive measurement of local business conditions in the vicinity of Providence would, if successful, form a unique contribution. Little or nothing had been done along that line in other quarters, but a member of the Economics Department was asked to interest himself in the problem. After a year of investigation and experiment, a preliminary series of four reports embodying certain preliminary findings was published in November, 1922, to February, 1923, by *The Providence Magazine*, organ of the Chamber of Commerce. Largely as a result of these articles, the Brown Business Service was proposed; its underwriting for the year ending June 30, 1925, was completed last summer, and the first report in the new series appeared October 1.

The bureau plans to issue ten regular monthly reports, dealing with economic conditions in Providence and vicinity as contrasted with those prevailing in New England and in the country at large. It also plans a supplementary series of reports on topics of special interest to local business men. The service is not intended to duplicate the work being done by other university or commercial organizations, but rather to supplement and support their work by translating general judgments into terms of an individual locality.

The bureau uses a great variety of factual and statistical information bearing upon local conditions. A novel feature among its statistical series is one representing the weekly volume of payroll disbursements in the various industries and trades about Providence. It is obtained by tabulating the withdrawals on payroll voucher from Providence Clearing House Banks.

Among other statistics in use are the following series, collected by the bureau on its own initiative or in coöperation with other agencies:

Employment in Rhode Island factories.
Industrial power sales of Narragansett Electric Lighting Co.
Cars loaded and unloaded locally on the N. Y., N. H. & H. R. R.
Cotton and wool consumed in Rhode Island mills.
Spindles and looms active in Rhode Island mills.
Value of building permits issued and floor space of contracts let in Rhode Island cities and towns.
Business failures in Rhode Island and the Attleboros.
Wholesale and retail sales in Providence.
Retail collections by Providence stores.
Retail prices in Providence.
Debits to individual accounts by Providence Clearing House Banks.
Commercial loans, discounts, investments, and deposits of Providence Clearing House Banks.
Deposits in mutual and postal savings banks of Providence.
Marriage rate in Rhode Island.
Number of telephone calls pegged in Providence.
Street railway passengers and revenues in Providence.

The Board of Directors includes the following: William H. P. Faunce, *ex officio*, President of Brown University; Thomas F. I. McDonnell, *ex officio*, President of the Providence Chamber of Commerce; Charles T. Howard, Chairman; Charles A. Horton, Vice Chairman; Henry B. Gardner, Vice Chairman; George W. Danielson, Paul C. DeWolf, Harry B. Johnson, Francis B. Keeney, Alfred B. Lemon, Arthur L. Philbrick, Theodore B. Pierce, and William L. Sweet.

The executive staff includes Professor Ralph E. Badger, Secretary-Treasurer, and Professor William A. Berridge, Director of Research. Other members of the department coöperate on occasion.

The North Carolina Institute for Research in Social Science.—The Institute for Research in Social Science, made possible by a recent substantial grant, began its work formally at the University of North Carolina on September 15, 1924. The purpose of the institute is the coöperative study of problems in the general field of social science, arising out of state and regional conditions. A board, consisting of the president of the university as *ex-officio* chairman and seven other members of the faculty appointed by the president, together with a secretary of the board, constitutes the governing body of the institute.

The principal activities of the institute are directed through the medium of research assistantships of which eight or ten will be available each year. To qualify as a research assistant, a graduate degree or the equivalent of one full year's research experience is required. Each assistant receives a yearly stipend of $1,500 and additional sum or sums for field expenses as the board may from time to time authorize. All appointments are for the term of one year; but, should the nature of the subject or the ability of the assistant warrant it, this term may be extended. These appointments are on a twelve months' basis, vacations and summer adjustments being arranged by those in charge of specific projects. Each research assistant devotes his entire time to his selected project with the exception that he may, with the approval of the professor in charge of his research, teach during any year not more than one course in a subject in harmony with his special field of investigation. He may also enroll for such graduate work as will not interfere with his research, and as may lead to the Doctor's degree, in instances where such a course seems advisable.

Two publications of the New York Association for Improving the Condition of the Poor are of especial interest to the American Statistical Association. The first, "The Growth and Development of Italian Children," by John C. Gebhart, deals with the problem of normal growth and development of children, being an intensive study of the weights and statures of 4,000 Italian children in a congested district of New York City, as compared with other dominant racial and social types. This study also includes comparative analyses of dentition, seasonal variations in growth and other factors bearing on the problem. The second carries the title, "Do Height and Weight Tables Identify Undernourished Children?" This study is a practical application of the material covered in the first publication referred to, to the problem of degree to which arbitrary height and weight standards can be exclusively relied upon as a method for selecting undernourished children. This report is the result of a joint study of the problem by John C. Gebhart, Director, Department of Social Welfare of the A. I. C. P.; Louis I. Dublin, Statistician, Metropolitan Life Insurance Co., and Dr. L. C. Schroeder, Associate Professor of Pediatrics, Cornell Medical College.

The Department of Labor of the State of New York has just issued Special Bulletin No. 126 containing an "Analysis of Workmen's Compensation Cases Closed July 1, 1922 to June 30, 1923." Formerly it has been the practice to base the tabulations on the year in which the accidents occurred, but by changing to the year in which the cases were closed, it has been possible to complete the tabulations for a given year earlier than formerly. A similar but more complete report, based upon the year ending June 30, 1924, is planned. It is also hoped that a bulletin on accidents statistics for the first ten years that the compensation law has been in existence will be prepared and published later on.

Among the statisticians visiting Canada and the United States as delegates to the British Association for the Advancement of Science and the International Mathematical Congress which met in Toronto in August were: Sir William Beveridge, Director of the London School of Economics; Dr. A. G. Bowley, Professor of Statistics at the London School of Economics; Dr. G. Udney Yule, University Lecturer in Statistics at Cambridge, and Dr. Corrado Gini, Professor of Statistics at the University of Padua and editor of *Metron*.

BRANDRETH SYMONDS

Dr. Brandreth Symonds, Chief Medical Director of the Mutual Life Insurance Company of New York, died at 33 Central Avenue, St. George, S. I. He had been a medical officer of the company for twenty-seven years.

Dr. Symonds was born at Ossining, N. Y., on March 4, 1863. Graduating from Hobart College in 1881, he received his M.D. from the College of Physicians and Surgeons three years later and in 1886 was house physician at Bellevue Hospital. Four years ago he received the honorary degree of LL.D. from Hobart College.

HENRY J. HARRIS

Dr. Henry J. Harris, Chief of the Division of Documents of the United States Library of Congress since 1911, died on October 10 at the age of 52 after a very brief illness. From 1902 to 1911 he was with the United States Bureau of Labor Statistics, winning recognition for special work in the study of labor legislation and workmen's insurance in foreign countries, subjects in which he was actively interested up to the time of his death. Dr. Harris was a graduate of Haverford College and received his Ph.D. in economics from the University of Pennsylvania in 1900. He was also for a time a student at the University of Halle. He has been a Fellow of the Statistical Association since 1909.

Dr. Raymond Pearl of Johns Hopkins University spent the month of June in London and lectured at University College on "The Bearing of Recent Experimental and Statistical Work on the Alcohol Problem." He also lectured before the Society of Biometricians and Statisticians in the Biometric Laboratory of University College on "The Curve of Population Growth."

Mr. Carl E. Parry, who has been recently engaged in a study of trading in wheat futures on the Chicago Board of Trade for the Department of Agriculture, has become a member of the Division of Research and Statistics of the Federal Reserve Board.

During the fall and winter the **Educational Committee of the American Statistical Association** expects to collect the facts on the teaching of statistics in American colleges and universities. The questionnaire is being drawn up and will be sent to the several colleges and universities. It is earnestly requested that teachers of statistics and the allied branches will supply the committee with the information requested. A progress report will be made at the annual meeting of the Association at Chicago.

MEMBERS ELECTED SINCE AUGUST

Andrews, William H., Kansas State Agricultural College, Manhattan, Kan.
Applegate, Melbourn E. S., Metropolitan Life Ins. Co., New York City.
Barber, Joseph H., Walworth Manufacturing Company, Boston, Mass.
Beckwith, Mrs. William E., College for Women, Western Reserve University, Cleveland, Ohio.
Bray, George A., Illinois Bell Telephone Company, Chicago, Ill.
Caldwell, H. Gibson, Queen's University, Kingston, Canada.
Cherniack, Nathan, Port of New York Authority, New York City.
Clark, William C., Economist, S. W. Straus Company, Chicago, Ill.
Eberlin, Arthur P., National Trade Extension Bureau, Evansville, Ind.
Flinn, Richard A., Chief, Div. of Employment, N. Y. State Dept. of Labor, New York City.
Greul, W. Herman, Consultant, 342 Madison Ave., New York City.
Hall, Keppele, The Joseph & Feiss Company, Cleveland, Ohio.
Hedrich, Arthur W., Dept. of Health, Chicago, Ill.
Lesher, Robert A., The Port of New York Authority, New York City.
MacKinnon, Joseph C., Registrar, Mass Institute of Technology, Cambridge, Mass.
Matthews, A. M., Comm. on Economic Research, Harvard University, Cambridge, Mass.
Morley, Burton R., University of Pennsylvania, Philadelphia, Pa.
Netusil, Francis Jaroslav, Private Docent of Demography on the Charles University Faculty of Science, Prague.
Strong, Wendell M., Mutual Life Ins. Company, New York City.
Washburn, Olive A., Western Electric Company, New York City.
Yasukawa, Kazutaro, Japanese Embassy, London, England.

REVIEWS

Canada's Balance of International Indebtedness, 1900–1913: An inductive Study in the Theory of International Trade, by Jacob Viner, Cambridge: Harvard University Press. 1924. x, 318 pp.

Economics is progressively becoming more inductive and statistical: the present work is a comprehensive inductive verification of the classical *a priori* theory of international trade. In general, it is a most painstaking study of the statistics of a particularly many-sided and elusive subject—the trade balance and its adjustment—which, including as it does the "invisible items" of trade, and especially the migration of capital, is become such a dominating factor in present day internationalism. The British statistics since 1921 have been annually assembled by the U. K. Board of Trade, following upon earlier estimates by Mr. C. K. Hobson.[1] A survey of still greater exhaustiveness has been made for the United States by the Harvard University Committee on Economic Research.[2] As the present review is being written, the Economic and Financial Section of the League of Nations is urging the different countries of the world to undertake annual analyses of their foreign trade balances, upon a prescribed plan, which includes a uniform balance sheet. Professor Viner's discussion of theory is thus in the forefront for timeliness.

During the decade ended 1913, Canada was the scene of one of the most remarkable expansions, relatively considered, that has taken place in that or any other country. The world over, indeed, that decade was a period of prosperity, as contrasted with the immediately preceding quarter-century of "bad times." Not to trench upon ultimate causes, it may be recalled that with the production of gold quadrupled and its accumulated stocks doubled, with agriculture once more upon a paying basis, and with population increasing at a rate that was probably unprecedented in the world's history, there began soon after 1900 a great forward movement to extend the areas devoted to the production of foods and raw materials. Capital in large amounts began to flow from Great Britain, till then the chief source of investment funds, in the direction of the "new" countries, notably Canada, Argentina, Brazil and India. France sent large sums to Austria, Russia and Turkey; Germany made considerable investments in Austria and America; whilst all the capital-producing countries, including the United States, raised and spent large sums upon internal development. Sir George Paish has estimated that in the five or six years following 1907, Great Britain alone exported 5,500 millions for investment, mainly in undertakings which had for their object the bringing into the international trade-scheme of new productive areas.

It was the inclusion of Canada in this movement, as a source of supply for raw materials, particularly cereals, that constituted the *raison d'être* of the

[1] *Economica*, May, 1921. See also his *Export of Capital*, London, 1914.
[2] *Review of Economic Statistics*, Preliminary Volume, No. 3. Harvard University Press, 1919.

great Canadian "boom" of 1900–1913. The immediate economic impulse was what was popularly called the discovery of the "last best West." The superficial phenomena were familiar enough. A spirited immigration policy drew thousands of people to occupy the new lands. An immense railway and town building program was begun, also primarily for the service of the West. The work of locating settlers, providing materials for their households, agricultural machinery for their fields, elevators for their crops, drew the entire economic life of Canada out of its accustomed orbit. The East turned to industrialism, and a revolution in consumption and distribution followed. It is the relative point of view that must be emphasized in the situation. Canada received approximately one-quarter of the entire capital exports of the British Isles during the period in question. Labor likewise flowed in more rapidly than in the case of any other country, the permanent population of Canada increasing 34.17 per cent and the gainfully employed population 52.7 per cent, between 1901 and 1911 alone. General productive activity showed a rise that has been estimated at 200 per cent. Wholesale prices went up between 1900 and 1913 at the steepest pace recorded in any country, 28.7 per cent; wholesale prices of foodstuffs advanced by 45.2 per cent.

It is the above history—the economic expansion of the Dominion of Canada at an extremely rapid rate during the years 1900–1913—that Professor Viner takes for background of his investigation. The choice is happily made. It may be said without exaggeration that to almost everything that happened economically in Canada during these years the heavy importation of capital furnishes the key: explain the trade balance of the period and you explain all. The statistics are fairly abundant (though not so plentiful as they have since become for similar phenomena), and they had been already assembled, rounded out and analyzed in a special official report well designed to facilitate such a study as the present.[1] Owing to the unprecedented scale of the capital importations, the records of all and sundry appertaining to the trade balance stand out with a sharpness that would be difficult, perhaps impossible, to duplicate elsewhere. It was this, of course, that primarily actuated Professor Viner in his choice of setting. On general grounds, however, as he points out, the geographical position of Canada, which decrees that a large amount of her producers' and consumers' materials enters into her international trade; her economic compactness, notwithstanding the disintegrating barriers driven by nature at three main points in her structure; her situation as between Great Britain and the United States, with their manifold influences upon her political and social habits; and finally her dependence upon New York for a gold market, which throws the mechanism of her foreign exchange into strong relief,—all these add justification to the selection. Though the treatment of Canadian affairs is essentially incidental to its theoretic object, the work will have a special value for all who are interested in Canada, on account of the light it sheds on a wide variety of conditions in the Dominion. One may regret that

[1] Volume II of the Report of the Official Board of Inquiry into the Cost of Living, 1914,—being a statistical examination of the economic causes of the rise in prices, 1900–1913; by R. H. Coats, member of the Board.

Professor Viner has not annotated certain of the permanent changes that have occurred in these conditions since the period of his review, though one must not quarrel with an omission that is expressly postulated.

The treatment falls into three main sections: (1) an introduction, in which the *a priori* theory is succinctly reviewed and the problem of verification set out; (2) Part I, a statistical investigation in several chapters of the invisible items of the Canadian trade balance during 1900–1913; and (3) an examination, also statistical, of the mechanism of adjustment of the Canadian trade balance during the period—the whole culminating in a review of the economic consequences of the movements that have been laid bare in the preceding. To each of these sections a brief comment may be devoted:

(1) Though the *a priori* theory of the mechanism of international trade has been comparatively immune from attack, three criticisms have been levelled at it *a priori*, namely, its dependence upon the validity of the quantity theory; its lack of precision as to the part played by gold movements; and its lack of definiteness when depreciated currencies are involved. Whilst these are singled out for special treatment, the study aims at a thorough inductive test of the entire classical explanation. Professor Viner recognizes of course the limitations of induction, and he analyses with acuteness the part played by the two methods in the subjective and objective phases of his thesis respectively:

> In the theory of international trade there must be left to the deductive theory, as its exclusive field, the exposition of why men act as they do in their international transactions, and of the subjective results of such action. The remainder of the theory is available to the inductive method, provided the facts have been collected and recorded. Induction can be utilized generally in coöperation with deduction, to describe and explain the changes in the volume and character of trade, the movements of price levels, the part played by exchange rates and gold movements in the adjustment of trade balances, and the reaction upon internal economic conditions of changes in international trade. The quantitative determination of the division of gains from international trade appears to be a problem beyond solution by either method. But quantities which are incapable of direct measurement are sometimes subject to measurement of their relative changes. The inductive method can be utilized here, both to test deductive conclusions as to the direction of the change in the division of the gains following upon a specific change in the conditions of international trade, and, given the direction of the change, to supplement this by an objective measurement of the degree of change in the ratio in which the gains are divided.

(2) For the working statistician, the immediately interesting part of the book is Part I, which is an analysis *in extenso* of the invisible items of the balance of trade, ranking with the work of Hobson and the Harvard Committee in translating Giffen's classic essay of 1882 into the terms of twentieth century conditions. In 1914 it fell to the official duty of the present reviewer, in the report already noted, to examine somewhat comprehensively the general economic situation in Canada, with more particular reference to the rise in prices which had taken place since 1900 and which in the prevailing unemployment had become the chief object of public complaint. Among numerous by-paths of the investigation was an analysis of the trade balance for the discovery of the imports of foreign capital as the fundamental force in the disturbance; and the estimates then made, though based in large part upon accepted formulae, have

remained the chief essay of the kind for Canada until the appearance of the present work. It is a pleasure to bear witness to the more thorough nature of the later investigation and to subscribe to most, if not all, of the conclusions now reached. Certain refinements will be possible in future official surveys which it is proposed to carry out. It would seem advisable, for example, to make additional eliminations from the grand totals of exports and imports before striking the commodity balance. For the value of the tourist trade to Canada, as well as on such points as the capital accompanying immigration and emigration, more mature judgments can now be obtained, whilst on points like freight and insurance payments the Canadian departments are in possession of information that was lacking in 1914. The direct statistics of foreign investments and of Canadian interest payments abroad are also steadily increasing. These, however, are details, and they have reference rather to the present and future than to the past.

In the final analysis, Professor Viner places the importations of capital into Canada during the period 1900–1913 at 2,506 millions, as against 2,713 millions of the earlier estimate, and his statement is probably as accurate as it can be brought without an excessive amount of further research. As an exposition of method the work can be heartily commended.

(3) In Part II, Professor Viner treats of the adjustment of the general price level and of the currency and banking systems of Canada to the above-mentioned heavy importations of capital. In other words, he shows how the capital represented in the adverse trade balance entered Canada in the form of goods. It is in this section that he has especially broken new ground, and if his treatment is still short of finality, he has organized materials and outlined a method that will be permanently valuable. His chapters on Canadian exchange and gold movements and on the operations of the Canadian banks in the adjustment of the trade balance are a clear exposition of a process that is popularly but little understood even in Canada, especially in its theoretic aspect. The gold movement into and from Canada is a matter wholly of banking and currency, the cash reserves of the banks and the government reserves against Dominion notes being its leading index. Professor Viner's illustration (pages 178–9) of a typical banking transaction in this connection is particularly clear.

Of more general interest is the discussion of price changes in relation to the balance of foreign indebtedness. Two chapters are devoted to this topic. In the first a complete restatement is given of the *a priori* theory, from Thornton, Malthus and Ricardo, through Mill, Cairns and Walker, to Taussig and Hollander. A following chapter is given up to the inductive verification. The problem of the latter is an exceedingly difficult one, for more than approximate mathematical correspondence between the two series of phenomena is not to be expected. In the Canadian official report that has been mentioned, the explanation of the rise in prices was laid mainly at the door of "industrialization" (including in the term the building of new railways and public works, as well as of industrial plant), and the consequent diversion of labor from the production of foods etc., to the production of "equipment," the international

situation being examined in this connection as well as the variation in "sectional" prices and the effects on the machinery of distribution. Professor Viner finds a more adequate explanation in the *a priori* theory of the trade balance, his principal objective being to establish that prices rose in Canada as compared with other countries in rough proportion to external borrowings. His explanation is Mill's as follows:

> The extension of loans to foreign countries would result immediately in an export of gold, with a consequent fall in prices in the lending country and rise in prices in the borrowing country. This change in relative price levels would stimulate exports, check imports, in the lending country, and in this manner there would occur an excess of exports over imports equal to the loans abroad. (Page 191.)

Mill's explanation is a theoretical and technical explanation, sound so far as it goes, and also general in its application. The explanation of the official report is a specific and practical application of the general principle to the Canadian situation, giving reasons why the enormous importation of capital between 1900 and 1913 was required, the purposes for which it was used, and the direct effects with which its use was followed in Canada.[1] This explanation in reality goes back of Professor Viner's to the causes of the borrowing. It is true, as he states (pages 249, 250), "there is no reason to suppose that prices would have been less buoyant in the borrowing countries if the loans made to them had been spendthrift loans." As a matter of fact, however, loans for railway building and other development purposes *are* "spendthrift loans" in their immediate effects, in that they involve consumption long before they influence production. Professor Viner moreover does not seem to attach sufficient importance to the fact that, in Canada at least, immigration of labor occurred on a great scale contemporaneously with the importation of capital. Immigrants in the four fiscal years ended 1911, 1912, 1913 and 1914, aggregated more than 20 per cent of the whole 1911 population, which, since immigrants are mainly adults, represented an addition of much more than 20 per cent to the preëxisting demand for consumer's goods—foods, clothing and housing. It is the history of all new countries that incoming settlers must consume before they are able to add to production, and this situation existed to a pronounced degree in Canada in 1911–14.

[1] The report sums up as follows: "Broadly, the rise in Canadian prices is shown as an integral part of the great expansion through which the country passed in the period 1900–1913, and the form which that expansion assumed, — the opening of the West as a feeder to the world cereal market, and consequent attraction of local energy from the supplying of local food needs, etc., into railway building, municipal growth and industrialism, — the whole being induced by the buoyancy of world credits which produced relatively similar price movements in certain other countries." Again: "The great rise in prices that has taken place in Canada is accordingly found to center largely in the new distribution problem that has been created by the lessening of local food supply during an era of heavy expenditures on capital account." It adds: "The incident is not new to Canadian history; almost an exact counterpart is to be found in the experiences of the provinces during the 1850's while the original railway and canal systems of the country were under construction." A footnote to the latter statement amplifies it with some interesting inductive data which Professor Viner might have found it worth while to examine: "Altogether about 100 millions were spent on railways in Upper and Lower Canada between 1849 and 1859, and an additional 11.5 millions on canals, about the same per capita as in recent years. . . . There was a real estate boom of enormous proportions, and a rapid growth of the cities at the expense of the rural districts. Immigration, foreign trade, and government and municipal borrowings were on an unprecedented scale. Prices rose 80 per cent in four years. Then, as recently, the rise was chiefly in native products. . . . The 'extravagance of the age' and the exactions of middlemen were frequent themes. The check came in financial stringency."

There appears no flaw, however, in the theoretic argument of Professor Viner, and it would undoubtedly have strengthened the Canadian official report had the *a priori* theory of the trade balance been more fully noticed, as in point of fact "capital importation" and "industrialization" in the wider sense of the latter term were regarded as synonymous throughout. The least satisfactory part of Professor Viner's induction lies in his correlation of conditions in this respect as between Canada and the United States. The price rise in the latter was, at the very least, two-thirds that of Canada's, ranking with that of the leading countries. On the other hand, net capital imports into the United States were but 1,000 millions, considerably less than one-half those of Canada absolutely, while relatively of course their importance was so much less as to cease to be comparable. It is scarcely possible therefore to argue that "on *a priori* grounds there should be expected a *moderately* greater rise in prices in Canada than in the United States." On the grounds of capital imports the rise should have been very much greater. Professor Viner covers this by a reference to the "self-sufficiency of the United States, which probably makes its price level more independent . . . and therefore more likely to move in response to purely domestic conditions." But this is to fall back on something very like the line of the Canadian official report.

The part of Professor Viner's concluding chapter, in which he sketches the general economic consequences to Canada of her borrowings during this period, will to a Canadian seem inadequate to so large a theme. That railway building was overdone, the history of subsequent administration abundantly proves; and there was undoubted overexpansion on the part of many municipalities. But that the latent strength of the country was seriously overestimated is not apparent. Viewed in physical terms the country is much wealthier today than it was in 1913: the acreages under crop have increased by 60 per cent; wheat production has risen to 474,199,000 bushels in 1923, a standard which has made Canada the leading wheat-exporting country of the world; whilst episodes like the discovery of the gold mines of Porcupine are of significant augury. As a test of financial strength the raising within Canada of nearly two billion dollars in war loans is worthy of mention, as is the fact that the life insurance in force has increased between 1913 and 1923 from $1,168,000,000 to $3,433,000,000. As regards the future, the rate of interest on the present external obligations must decline with the replenishment of the reservoirs of the world's capital, whilst the prices for Canadian exported food products are likely to continue profitable with the restoration of Europe and the delayed recovery of Russia. The prospects for Canada may reasonably be viewed without pessimism.

As already indicated, the work is an admirable example of the union of the two great methods of economic reasoning, after a model that has become increasingly prevalent and necessary. Not the least of its benefits will be the spur it administers to official statisticians for more competent data on points of such vital interest. It needs but to mention Mayo-Smith to recall how much of economics is capable of similar treatment.

R. H. Coats

Ottawa, Canada

Insurance by Industry Examined, by Joseph L. Cohen. London: P. S. King & Son, Ltd. 1923.

Mr. Cohen's treatise deals with unemployment insurance in Great Britain. In his five chapters the author discusses the growth of the British plan of unemployment insurance; the arrangements with trade unions and the conditions governing supplementary schemes; criticisms of the government scheme; unemployment insurance by industry; unemployment insurance as a part of social insurance. There are three appendices: (1) unemployment insurance abroad; (2) Professor Commons' "unemployment compensation and prevention scheme," which is described as "a highly ingenious and well-thought-out plan for avoiding the special difficulties in the way of unemployment insurance legislation in the United States"; (3) administrative costs of schemes of social insurance. A selected bibliography is included.

To the enthusiast for any marked change in the British plan of unemployment insurance the book will be disappointing. Mr. Cohen stands for the *status quo* in legislation at least, although he finds criticism enough. Some of those who heard Sir William Beveridge at the recent meeting of the British Association in Toronto felt that he rather disparaged criticisms and alternative suggestions. Cohen states that "the existing scheme of unemployment insurance in this country has been subjected to a constant fire of criticism during the last two years."

To the charge that the cost of administration is unduly high the author answers that the percentage of administrative costs to the sums disbursed in unemployment benefit is about 8.8, while the cost of workmen's compensation as administered by proprietary insurance companies has been 50 per cent, of industrial insurance 44 per cent, and of fire insurance 50 per cent. He asks those who cry "bureaucracy" to explain in what way the system is more bureaucratic than large private corporations. Whether or not the staffs are too large is a matter, he states, that still remains to be investigated. Cohen finds no evidence to support the accusation that undeserving applicants receive benefit and he counters with the significant assertion that no one would appear before the Departmental Committee of Enquiry into the Work of the Employment Exchanges to substantiate the charge.

It has become the fashion with opponents of unemployment insurance in Great Britain, and it has been adopted by many here, to speak of the Fund as bankrupt and of the benefits as "doles" thoroughly demoralizing to the unemployed. Cohen makes a spirited attack on this form of depreciation. He asserts that if the number of unemployed should be reduced to one million, benefits could be paid from current income and the debt would soon be cancelled. This forecast is well on the way to fulfillment for the Fund has been paying its obligations to the Treasury so rapidly that Mr. Shaw, the present Minister of Labour, felt justified in advancing the rate of benefit and doubling the rate of allowances for dependent children in the new legislation just passed. Cohen argues with the writers of *The Third Winter of Unemployment* that there is no evidence of demoralization resulting from provision of maintenance without work. It is an abuse of terms to call unemployment insurance benefit, towards

the cost of which the worker has contributed, "a dole." "To receive what they have a right to is not demoralizing unless, indeed, this atmosphere of charity is made to surround it." One is reminded of a remark in the House, made I think by Mr. Clynes, to the effect that the benefit resembles a dole only in the dictionary definition that it is something given grudgingly. As the author states, the growth of trade unionism, of the Labor Party, and of the coöperative movement prove that English workers have not ceased to look to their own organized efforts.

While these criticisms are repudiated the authorities are taken to task for their lack of attention to prevention of unemployment and it is stated that there have been "very serious omissions and errors on the part of those responsible for the legislation bearing on, and the administration of, unemployment schemes." There is general agreement that the scale of benefit is far too low (Mr. Shaw's Act slightly increases benefits) and the flat scale is objectionable in that all must receive the benefit provided for the lowest paid workmen, otherwise those in the lower wage levels would receive more benefit than wages. In the author's opinion "a graduated scale of benefits is therefore urgently called for."

Poor law relief as a supplement to unemployment insurance is condemned and it is contended that benefit should be raised to such an extent that outdoor relief can be confined to its proper sphere of aiding cases of exceptional distress. Poor relief aids only those in need and so differentiates in favor of the man who does not save and against the man who does. The author finds a gratifying absence of distress despite the unprecedented unemployment. Orderly government would have been jeopardized without the national scheme and he concludes that criticism has been directed at details, as a rule, or to the development of alternative schemes. "The fundamental device of insurance against unemployment is generally accepted."

The fourth chapter, on Unemployment Insurance by Industry, is perhaps the most interesting. The author finds that the proposal has made headway among employers and employees in recent years as part of the idea of industrial self-government. In America the term "insurance by industry" implies insurance by several employers or by an individual establishment or by employers and employees, as against any government plan. In Great Britain the proposal is less definite. It is not clear whether industry will pay the whole costs and forego the government's contribution or whether the state subsidy will be continued; whether each industry will have its own employment exchanges or whether the national system will be retained; whether each industry will finance its own unemployed as part of a national scheme with different rates of contribution for different industries; or whether each industry may finance an autonomous scheme. Some are advocating that trades which satisfy specified conditions should be permitted to establish their own schemes, apparently unaware that it is possible to contract out of the state scheme.

These difficulties aside, others present themselves. The problem of defining an industry would arise and definition would involve friction with the unions. Some labor would not be assignable to any industry and the necessity for the

workers to pass between different systems of insurance would reduce the mobility of labor. The most serious problem would be the devising of suitable administrative machinery, able to disburse benefits and provide a test of unemployment. Even if exchanges were established for each trade, the government exchanges would still be necessary for the common pool of unskilled labor. Further, no industrial scheme could be introduced in a period of abnormal unemployment when claims exceed the normal sources of income. To one of the main arguments for insurance by industry, that the charges should be commensurate with the rate of unemployment, Cohen admits that a flat rate contribution and the same scale of benefits for all is open to criticism, but he holds that the state subsidy makes it worth while for the more regular trades to pool their risks with the others; and also that it would be possible to vary the amount of benefit in some relation to the normal wage of the worker under the existing plan as well as under any industrial scheme. After the insurance trade contracted out of the general act and established its own scheme, benefits were increased and contributions by employees eliminated. The author dismisses this as "a very exceptional instance" but one cannot help thinking that other industries of a similar character would do equally well under their own arrangements.

Answering another main argument for unemployment insurance by industry, that the provision of premiums varying with the rate of unemployment would stimulate employers to reduce the amount of unemployment, Cohen states that the major causes of unemployment, the cyclical fluctuations in industry, are beyond the control of a single industry. He argues also that no incentive any scheme can devise can begin to equal that already operating through wasting plant, overhead expense and loss on accumulated stock. He further reminds us that the inducements offered to the employers by the Act of 1911 to furnish regular employment had so little effect that they were withdrawn. The individual manufacturer might regularize employment to some extent but he doubts if a whole industry could achieve much along these lines.

As to the indirect benefits arising from joint control of unemployment insurance by industry, the author believes that these may be derived from the present scheme by giving those most directly affected more control of appointments and even of policy. He concludes that:

> Insurance by industry would be a very troublesome and costly method, involving very many difficulties at the outset, that it has no direct advantages over the state scheme, and that the indirect benefits, which are not very great, can be achieved by a slight amendment of the existing scheme. The Unemployment Insurance Act of 1920, slightly amended, should be tried for a decade before any fundamental revision is attempted.

In his final chapter the author considers the demand for a unified scheme of social insurance. He examines the plan proposed by the Rt. Hon. C. A. McCurdy to cover health insurance, unemployment insurance, workmen's compensation, old age pensions and pensions to widows and orphans. He outlines a scheme worked out with particular reference to the mining and to the boot and shoe industries. McCurdy avoids all discussion of administration, but

Cohen considers whether administration should be entirely by the state or partly by the state and partly by private companies. While he concludes that government funds have smaller administrative costs, are more prompt in paying benefits and are not charged with the abuse of lapsed policies, as are the private companies, the fact remains that the administrative expense of government schemes has been criticized by the Webbs and others. Accordingly the question is raised whether a general scheme of social insurance could not be non-contributory and so less expensive. In short, that insurance, like education, should be free, as Pigou suggests. From a long-period point of view it matters little who actually pays for social insurance, but the continuance of contributions may be desirable so that the two parties may have a greater interest in the expenditure of the money by the government.

The author suggests as one plan for a non-contributory system of social insurance the establishment of a Department of State, charged with instituting schemes for reducing the hazards of the worker's life. There would be a social insurance office, perhaps the present Employment Exchange, in every village and the worker would deal with this one agency. The Treasury would provide the costs, as it does for education, although a strong case can be made out for an additional tax on industries whose rates of unemployment and accident are above the average. Further enquiry is suggested, for a completely revised code of social insurance is needed. "There is widespread discontent with the law as it is today."

The book is useful. It makes possible an understanding of the British scheme of unemployment insurance without consulting a pile of blue books. The final word, which amounts to an injunction to "wait and see," and that for a decade, is not easily accepted at first. But what more can be suggested? Practically the whole industrial population has been brought under the scheme and a vast machine has been built up. This country's experience with the United States Employment Service proved that years are needed to perfect such an organization and the wisdom of making haste slowly is obvious. Perhaps if the present beginnings in unemployment insurance by industry in the United States have any considerable development it may become necessary to defend the established order against overhasty advocates of an all-inclusive government scheme.

But what trends will be revealed during the decade? Already Mr. Shaw's Act has granted slightly more generous treatment and uncovenanted benefit has been made a legal right equally with covenanted benefit. Will the movement for self-government in industry influence the scheme more strongly with the passing of the abnormal period? Will the state system hold the field or will its sphere ultimately be that of serving the unorganized industries and of coördinating the various industrial systems? We can only say with Mr. Cohen, "wait and see."

BRYCE M. STEWART

Unemployment Insurance Office
Men's Clothing Industry
Chicago, Ill.

Sugar in Relation to the Tariff, by P. G. Wright. Institute of Economics Series, Investigations in International Commercial Policies. New York: McGraw-Hill Book Co., Inc. 1924. xiii, 312 pp.

As a presentation and analysis of the facts upon consideration of which tariff policy with respect to sugar should be based this is a most excellent piece of work, and no degree of praise would be too high. Its survey of the technological and commercial aspects of the sugar industry is at once compact and comprehensive, and in spite of his feats of compression the author probably gives a more complete and better-balanced study of the industry than is to be found in any other one source. In so far as the fundamentals of the tariff problem are concerned, the author further makes some important contributions to tariff technique. But on the basic question of tariff policy he is, to my mind, wholly and inexplicably unsatisfactory.

Mr. Wright demonstrates conclusively enough that the objective of Section 315 of the Fordney-McCumber Tariff Act, namely, the "equalization" of foreign and domestic costs, is chimerical, since marginal costs, after allowance for freights, tariffs, etc., always tend to be equal, and an equalization of average costs through a change in tariff rates will soon bring about new arrays of costs in both countries such as once more to equalize marginal costs and such as to make average costs no nearer equalization than they were prior to the tariff change. It should be pointed out, however, that this will be true only if there is both domestic production and importation of the commodity concerned both before and after the tariff change. He argues persuasively that the real function in tariff technique of cost of production comparisons is to serve as a basis for the determination of the tariff rate necessary to bring about the desired proportions between the domestic and the foreign contributions to domestic consumption. He shows that even under free sugar a substantial proportion of the American consumption would be supplied from American, including colonial, sources. He presents convincing evidence in support of his claim that the present tariff rate on Cuban sugar is unnecessarily high to maintain the present approximately equal apportionment of the American consumption between American and foreign sugar. He estimates that the present tariff on sugar results in a net cost to the American public, over and above the customs revenue produced thereby, of $92,000,000 per annum.

The statistical data from which the author derives these conclusions, although they were unquestionably the best available, fail on two grounds to meet the strict theoretical requirements. The cost data used are all exclusive of interest. Differences as between the different producing regions in the amount of capital utilized per annual unit of sugar produced and in the prevailing rates of interest might conceivably have been substantial enough appreciably to affect the quantitative conclusions reached. Secondly, the range of costs for the different portions of the output of each producing region is determined from the cost data for single factories or for groups of factories under single management. These data presumably are derived from standard accounting records and are the average costs for each factory or group of factories. The margin is thus located at a "point," namely, *the* "marginal producer," whereas the

economist would ordinarily contend that there are different costs for different portions of the output of each producer and that every producer tends to be a marginal producer with respect to a portion of his output. A theoretically satisfactory method of determining the amounts of sugar which the American industry could profitably produce at any prices in a hypothetical price series would take into consideration the range of costs within the individual enterprises and would aggregate these, rather than basing its conclusions on the range of average costs as between different producers. If the results of the two methods were in even approximate agreement, such correspondence would appear to be accidental rather than the consequence of mathematical necessity.

It may be parenthetically noted that the author departs from the traditional use of the term "marginal cost" to signify the highest cost at which any of the output is regularly produced, by applying it to the cost which exactly equals the price, or what has recently come to be known as the bulk-line cost (page 114). He nevertheless adheres to the traditional formulation of price-determination theory, although it is sapped of all but tautological meaning when marginal cost is used to signify bulk-line cost. He states that "long-run economic forces always tend to equalize the marginal cost and the price" (page 117). Given his definition of marginal cost, why "long-run"? Why not also short-run? Why "tend"?

The author is limited by certain self-imposed limitations in deriving conclusions as to tariff policy from his study, but these limitations deprive the conclusions of any value. It is, moreover, far from clear whether the conclusions are really offered wholly subject to these qualifications. The conclusions are said to be based on the assumption that

> The United States is to continue its historic policy of maintaining a customs tariff policy as a part of its fiscal system and that the tariff will be used in the future as in the past as a means of raising revenue and also as a means of affording protection to domestic industries. With this assumption the problem becomes definite and concrete. It is to determine: (1) Whether, after giving due consideration to all the conflicting interests concerned, sugar should be dutiable or on the free list. . . .

There is no intent "to undertake an abstract discussion of the merits of protection as a principle of national policy nor of the merits of a customs tariff as a method of taxation" (page 5). How can one "give due consideration to all the conflicting interests concerned" if the validity of the claims of one of these sets of conflicting interests has already been assumed without argument? And if there is no intent to enter into "abstract discussion" of the merits of protection, Chapter VIII was written and included by oversight, for it sets forth its objectives as follows:

> In this chapter will be presented the leading arguments for and against a duty on sugar. *Some of these are of a comprehensive character and relate to commodities in general. Such arguments necessarily raise the whole question of the wisdom of maintaining a customs tariff for the purpose of protecting a domestic industry.* Other arguments relate specifically to sugar (page 213, italics mine).

Nor is there clear evidence that the author was really subject to any limitation when he reached his final finding that: "Such a dispassionate study and weighing of claims leads to the conclusion that a *wise* national policy calls for the retention of a duty on sugar" (page 248, italics mine).

Interpreting as best I can the intent of the author in the light of his mental reservations, I understand his main conclusion to be that, given the present American protective tariff policy, whether it be good or bad, it is better with a sugar duty than it would be without. There is so sharp a contrast between the decidedly free-trade trend and manner of the analysis of the facts in the body of the book and the protectionist trend of the conclusions, and there is also so striking a difference in the intellectual quality of the two portions, that it is hard to believe, what one must perforce do, that they are both the products of the same hand. The arguments marshalled in support of the retention of a duty on sugar are feeble. The government, it is said, is hard pressed for revenue and sugar is admirably suited for the purpose. The government, however, is not hard pressed for revenue and has been reducing its taxes. It has not been generally supposed by economists that protective duties were admirably suited for revenue purposes. If the revenue from the tariff is indispensable, are there not other commodities, notably coffee, which would be clearly superior to sugar from the fiscal point of view? The burden of a sugar duty, says the author, would be widely diffused. So would be the burden of a duty on coffee. So would be the burden of a poll tax, or any other regressive tax, but that is scarcely to be accepted, without further argument, in its favor. The author proceeds to plead the vested interests of the sugar-producers as justification for the retention of the import duty. I had supposed the vested interests argument had long lost its social standing in respectable circles, except as an argument against the sudden elimination of protective duties of long standing. Why not gradually proceed to the establishment of a vested interest of the American consumer in freedom from tariff exploitation by a favored and by no means indigent group? The sugar industry, says the author, "is a desirable industry to maintain because of its beneficial effect upon the soil, improving it for other crops and depleting it of no valuable mineral constituents." If in spite of these beneficial effects the farmer will not raise sugar without tariff protection, is it not evident that the benefits are not sufficient to offset in the estimate of the farmer the superior economic attractions of other crops? It is possible, of course, that the farmer errs in his calculations, but has the author even presumptive evidence to that effect at his command? Also, says the author, "there may be some wisdom in maintaining a strong domestic sugar industry in the interest of national self-sufficiency." It is regrettable that this argument was not given extended development, for an attempt to demonstrate that a larger domestic sugar industry than would survive free sugar is important to the economic or military or cultural well-being of the United States would have made interesting reading, especially as applied to the sweated child-labor of the sugar-beet fields.

From all of which application of science to politics there results the valuable conclusion that the duty on Cuban sugar should be reduced from its present level of 1.7648 cents per pound, but should be maintained at somewhere between 1.25 and 1.50 cents—at an annual net cost to the American public, according to the author's own method of calculation, of about $70,000,000!

JACOB VINER

University of Chicago

Portland Cement Prices, by Henry Parker Willis and John R. B. Byers. New York: The Ronald Press Co. 1924.

The Portland cement industry has recently been very much in the public eye. In December, 1923, certain of its members were adjudged conspirators in restraint of interstate trade by a United States district court. Among the many charges filed against these cement manufacturers was the fixing of uniform prices. In recent months has appeared a volume *Portland Cement Prices* which is "intended to explain the way in which [Portland cement] prices are determined, changed, and regulated by economic conditions." The authors are Henry Parker Willis and John R. B. Byers.

Our annual production of Portland cement has increased more than one hundredfold since 1896. The 115,000,000 barrels—approximately half a billion bags—manufactured in 1922 was the product of 120 separate plants, owned by 90 different companies. About 90 per cent of these companies are members of the Portland Cement Association with general headquarters in Chicago and offices in 27 other cities. The protection and preservation of the welfare of the cement industry is the ostensible end of this association.

From 1902 to 1915 the national association was the most important body in the Portland cement industry. Among its many functions were the development of uniform cost accounting systems, the formulation of uniform trade practices for the industry, and the collection and distribution of statistical data bearing on Portland cement. By 1915 the industry had become so large and scattered that it was determined to form local or sectional organizations to take over certain of the duties of the national association, and thus leave the latter free to devote its attention to matters which affect the interests of the industry as a whole. Such duties consisted primarily of collecting, compiling and distributing statistics and other data which were of particular interest to the mills located in a given territory. Accordingly, five sectional organizations were formed with headquarters at New York City, Chicago, Dallas, Kansas City and Atlanta respectively.

In the course of collecting and disseminating statistical and other data certain of the sectional organizations—in some instances called audit and statistical bureaus—secured from each of their members or "subscribers" various types of information. Among such information were:

(a) Daily reports of quotations to customers, including among other things, the name of the party to whom quotations were made; the destination of shipments; number of barrels specified; prices, f. o. b. destination; the expiration date of quotations; and a description of the work to be done.
(b) Daily sales reports with name of customer, destination, and prices, f. o. b. destination.
(c) Daily shipping reports.
(d) Monthly reports on shipments into a given territory and monthly reports on production and stocks on hand.

In return some of these local associations or bureaus sent to their members or "subscribers" monthly summaries of the total shipments by all subscribers into a given territory and summaries of total production and of total stocks on hand.

This practice of reporting sales and quotations, it has been held by the U. S. Department of Justice, had, in certain instances, made it possible ("and in fact it is customary" [1921]) for subscribers to telephone to the local statistical bureau offices and ascertain whether other subscribers had submitted quotations on a particular job; and, if so, the prices quoted.

In addition to exchanging statistical information members of certain local associations (bureaus) maintained arbitrary freight basing points, from which freight was charged, regardless of the point from which cement was actually shipped. The bureaus distributed to their various members complete tabulated lists of freight rates from the different basing points to all points in the territory in which members market their products. Members established mill base prices for each basing point and all sales were made f. o. b., point of delivery, requiring purchaser to pay the freight and allowing credit for such freight charges when he made payment for his merchandise. Such practices made for uniform prices.

In 1921 the Attorney General of the United States brought suit against the local cement associations and statistical bureaus, charging them with being unlawful combinations and conspiracies in restraint of interstate trade. Among the many results following such combinations, he mentioned "uniform prices to contractors at the same point of delivery, notwithstanding differences in cost of production and freight rates." As described in a petition to the District Court of the United States, Southern District of New York:

> A dealer located at, say Brockton, Mass., requesting quotations on a quantity of cement for stock, has substantially this experience: If he is a recognized dealer as defined in the Trade Practices, if the manufacturer thinks the dealer can make proper use of the quantity of cement specified, if the reports of defendant Eastern Association show that the particular dealer is not indebted to any other cement manufacturer, although the indebtedness may be the result of a refusal to pay because of a legitimate controversy rather than a lack of ability to pay, the manufacturers will quote. Upon examination of the quotations the dealer finds that all companies quote the same price regardless of the location of their mills and the distance from Brockton, that all prices are f. o. b. Brockton, that all companies require him to pay the freight and permit him to deduct the amount thereof in making payment for the cement, that all companies allow him the same credit for bags returned in good condition, that all companies allow him the same discount if the bill is paid within 10 days, that all companies quote for immediate acceptance and for only one carload, which must be shipped within 15 days.

On December 13, 1923, United States District Judge J. C. Knox adjudged one of the larger of the five regional Portland cement associations (Cement Manufacturers Protective Association with headquarters in New York City) an unlawful combination in restraint of interstate commerce and perpetually enjoined it and prohibited its members from making or receiving such reports as were described above and from doing numerous other things—among which were the establishment of uniform mill base prices and freight basing points and selling Portland cement f. o. b., point of delivery exclusively. Hence were eliminated by court decree many of those conditions which made for uniform cement prices.

In February, 1924, H. Parker Willis and John R. B. Byers undertook "at the

request, and in behalf of some leading producers in the eastern districts of the United States" . . . "to explain the way in which (Portland cement) prices are determined, changed and regulated by economic conditions and to apply tests which may serve to prove the basis for a judgment of their character" (page 3). "The final expressions of opinion presented are those which have been independently reached by the writers" (Preface).

After stating that cement prices tend to be uniform and are subject to broad changes only at infrequent intervals, the authors set out to learn why this condition of uniformity exists. They find the reason in the nature of the product, a highly standardized commodity, sold to a group of specialized buyers.

> Selling this standardized commodity to a group of specialized buyers requires a sales force of men who are continually combing their respective territories. A corps of employees furnishes the salesmen with a list of new prospects, the dealers' past purchasing record, prospective contracts about to be awarded, and all sales information. These salesmen are continually coming into contact with the various dealers, who usually handle several brands of cement, and are securing information as to the prices offered by competitors and other field developments. This information is immediately sent in to their executives. Such contact acts in the same way as the "ticker tape" of the stock market, and since no one can claim any material advantage for his product, it tends to produce uniformity of price (page 20).

These uniform prices are quoted f. o. b. destination, for, as say the authors, "since purchasers of cement want to know the cost at the place where they propose to sell or use it, manufacturers have assumed the inconvenience and difficulty of furnishing prices, including delivery" (page 23).

Thus do uniform prices come about in the Portland cement industry. Indeed, this uniformity is to the authors in itself an evidence of perfect competition. They quote from Professor E. R. A. Seligman and other eminent economists: "In no instance where there is a perfect competition can there be more than one price for the same thing at the same time." (Had they turned to page 239 of Professor Seligman's *Principles of Economics* [ninth edition, revised] they would have found that this same uniformity is true of monopoly prices. Says Professor Seligman: "But as the seller has control of the supply, there is only one supply price." . . .)

Having so disposed of the question of uniform cement prices, the authors raise the query: Are they fair, reasonable and equitable? They have collected a mass of statistical data dealing with cement prices and cement profits and in the light of certain tests attempt to answer these questions. Fair prices, they define as those which "return to the producer the current or ordinary income properly resulting from the service rendered by commodity supplied" (page 11). Reasonable prices are those which "return only a normal income upon investment or outlay made by the producer" (page 13), and equitable prices are those which "apply similarly to all consumers under similar conditions" (page 10).

The authors' conclusions are to the effect that cement prices are fairly reasonable and equitable. Neither prices, earnings, or dividends indicate monopoly or combination. The uniformity in cement prices as between different companies, they say, is not greater than that which characterizes the price of

other standard commodities. (Comparisons are made with the prices of common brick and such monopolized or semi-monopolized products as gasoline, kerosene, petroleum and wire nails.) Indeed, at times "cuts or advances in price made by producers who for one reason or another are dissatisfied with existing conditions" (page 53) have made for disappearance of uniformity. The customary or normal return on the capital invested in the cement industry is in the neighborhood of 7 per cent, a return which does not "exceed about two-thirds or less of the average or normal return obtained by enterprises in all lines of business" (page 68). In short, if one is to judge "either (1) by prices for other commodities, of like sort, (2) by the general level of prices for all commodities, (3) by earnings or dividends, when studied in the light of general fiscal data, or (4) by utility or value as obtained by the consumer and as indicated by his use of the product" (page 69) he must conclude that if there is a combination or monopoly in the Portland cement industry, "it has not succeeded in inflicting upon the consumer an unreasonably or inequitable level of prices" (page 69).

And so closes the argument. Its cogency and value must be left to the reader who should peruse not only the book itself but also the petition and decree in United States of America *vs.* Cement Manufacturers Protective Association.

ISADOR LUBIN

Institute of Economics
Washington, D. C.

Workmen's Compensation, by E. H. Downey. New York: Macmillan Co. xxv, 223 pp.

This little book is made important by a most excellent series of notes.

While speaking of the cost of the insurance which workingmen have, Mr. Downey says: "These figures speak for themselves. The typical wage earner neither does nor can provide out of his own earnings against contingencies, accident, sickness, unemployment, or old age." This is true now as it was during Mr. Downey's life.

He truthfully says: "A really adequate scale of compensation benefits, such as has never yet been established in the United States or elsewhere, would add something like ten cents to the cost of a sixty dollar suit of clothes, thirty-five cents to the cost of a fourteen dollar ton of anthracite coal and two hundred dollars to the cost of a ten thousand dollar home."

As to the rate of payment, he says: "Seventy-five per cent of wages would better meet the prime requisite of comfortable support during incapacity and would, at the same time, sufficiently penalize voluntary absence from work. Even the 75 per cent rate, with a three-day waiting period, requires the injured workman to bear one-third of his wage loss during a disability of four weeks. Will anyone assert that less than two-thirds of actual wage loss is an adequate indemnity?"

"Commutations of future installments ought very rarely to be allowed. Few wage earners or their widows are able to make secure investments of consider-

able sums and fewer still can embark in business with any prospect of success." So says Mr. Downey in discussing mode of payment of benefits.

He also says that "when the price of a hand is set at $2,000 there is no great incentive to employ the best that modern surgery has to offer. But if compensation for the loss of a hand were a life pension at one-half of wages, neither employers nor insurance carriers would stint their efforts to save the industrial use of a threatened member."

"The beneficiaries of compensation are wage earners and their wives and children. The benefit payers are insurance companies and large employing corporations. In these facts reside both the power and the incentive to evade legal responsibility, withhold compensation and enforce unconscionable settlements." Thus Mr. Downey puts simply and clearly the reason for poor settlements.

"The indispensable means to these ends are effective supervision of settlements by agreement and appropriate tribunals for the adjudication of controversies. . . . But direct settlement between parties so unequal as a foreign-speaking widow and an international insurance corporation will too often mean oppressive settlement unless the rights of claimants are protected by the administrative arm of the state." In this way Mr. Downey suggests the only proper method of administration.

Here is his idea of a board: "But the board should retain supreme authority alike in the purely administrative and the quasi-judicial functions. All subordinates, from clerks to referees, should be appointed by and responsible to the board and the commissioners should interest themselves not less in administrative policy than in the decision of cases. The board should be irremovable except by impeachment and be paid salaries commensurate with the emolument of judges of the higher courts. The referees should have a like permanence of tenure and the whole administrative staff should be protected by civil service rules."

In discussing the subject of compensation insurance, Mr. Downey says: "Under a low scale, with payments running but five or six years at the maximum, many employers are able individually to assume the risk of loss and to provide out of their own resources ample security that future installments of compensation will be paid when due. But under any adequate scale of benefits payments will continue to the permanently disabled for life, to widows during widowhood and to children to the age of eighteen. The cost of a single death or permanent disability will frequently exceed $25,000 and pensions running for ten years and upwards will comprise far the greater part of all indemnity for work injuries. It is just these long continuing payments which are of the utmost social importance because upon them depend the nurture and education of whole families, and it is precisely these obligations that employers, individually, are least able to assume."

Concerning the reserve of workmen's compensation insurance companies, Mr. Downey states as follows: "Compensation reserves must, of course, be computed by states and under the supervision of state officials. The readiest and most effective means of segregation, therefore, is to require the deposit in each

state of securities sufficient to cover all liabilities outstanding in that state as ascertained by the supervisory authority."

Under the competitive system he foresees the necessity of supervision and says concerning this: "Regulation which seeks to restrain competitive abuses and extravagance without abolishing competition must control total premium rates, total expense loading, acquisition or selling costs and the classification and rating of individual risks. The administrative authority should determine the reasonable and adequate rate for each risk class and enforce the observance thereof by all carriers, leaving participating insurers free to return to their policyholders in the form of premium dividends any savings which they may severally effect in losses and expenses."

He states that "only six commonwealths and the District of Columbia retained the outworn system of employers' liability," and is of the opinion that "some two-thirds of the work injuries which annually occur in the United States are covered by the compensation scheme."

He discusses the insufficiency of American compensation, saying: "Amid this welter of inconsistency, the single feature common to all the acts, though in varying degrees, is insufficient benefits. No state has established a really adequate scale of compensation, such as would serve to minimize the social cost of work injuries," and finds that disability benefits are 50 per cent in twelve states, 60 per cent in nine states, 65 per cent in four states, and 66⅔ per cent in eleven states and the Federal Government. He finds that the weekly compensation is almost everywhere subject to fixed maxima—none at all in two, $12 in five, $15 in nine, $16 to $18 in fifteen and the Federal Government and more than $18 in only seven.

In total disability he finds but nine states pay a straight life pension and in five states the benefits are reduced if "the injured is so unfortunate as to survive beyond that period" and in all other states payment is limited to six, eight, or ten years and often to $4,000 or $5,000.

In the case of death benefit he finds but seven states and the Federal Government provide a pension to the widow during widowhood and to children up to the age at which they may legally be employed, and in four other states pensions for the children continue through the period of compulsory school attendance, although the widow's pension terminates six or eight years after the husband's death. In quite three-quarters of all the states the death benefit stops after six years, irrespective of the number, age or status of dependents.

Applied to children he finds it nothing but infamous that "a majority of the children orphaned by industrial accidents are under eight years of age, more than a third are under five and nearly a sixth are under two. Compensation to these children is, in most states, cut off before they have finished the grammar grades or even before they have entered the public schools."

Mr. Downey also condemns in equally open manner the failure of American compensation laws to provide such medical relief as may be required.

He finishes his most excellent book with the following prophetic words: "Adequate compensation benefits will be attained as fast as public opinion is educated to the necessity thereof. Already the earlier laws have repeatedly

been amended upwards and the process gives every promise of continuing for at least another decade. High benefits are the prime desideratum, for all things else will follow thereupon. High benefits will enforce curative treatment, retraining and prevention; with high benefits, also, the need of efficient administration and of sound insurance at reasonable cost will be irresistibly apparent."

MILES M. DAWSON

New York City

Some Aspects of Italian Immigration to the United States, by Antonio Stella. New York: G. P. Putnam's Sons. 1924. 124 pp.

The author of this book has for many years been known as a leading representative of the Italian strain in New York. He is a physician who early came to the attention of students of immigration by his articles on important relations between immigration and tuberculosis. In the book here under review, he disclaims any attempt, as his title indicates, to handle his theme comprehensively and seeks only to consider various matters which have come to be in dispute. His book is in fact principally an argument against those who lend themselves lightly to prejudice when they discuss immigration.

In popular and political writings our Italian immigration rather more than much of our other immigration has been the object of ignorant and biased criticism. Some of this criticism Dr. Stella counters by citing statistics drawn chiefly from our censuses and from the reports of the Commissioner General of Immigration; other parts he counters by reinterpreting statistics that have been misinterpreted or by drawing upon his own intimate knowledge of Italian ways and conditions.

He sees no virtue in naturalization accomplished by pressure and propaganda (neither does the reviewer). He detects serious flaws in the conception and application of intelligence tests and in the interpretation of their results. And it must be admitted that the tests signify less today than they seemed to many to signify in the first flush of the excitement which they provoked; a restatement of their import is required. To illiteracy, disease, criminality, Dr. Stella gives special attention. There are times, however, when his own interpretation appears to rest upon a false emphasis; for example, the low rate of illiteracy of the children of the foreign born as compared with that of the children of the native born does not mean that they "more readily appreciate its [this country's] school advantages than do the native born," but rather that the parents of these children have settled in the cities and towns, while more of the children of the native born are in the rural districts where schooling is less easily had. Dr. Stella devotes sections to the statistics of money shown at our ports and the question of money sent out of the country—it is a sad commentary on our times that it should still be necessary to argue these points.

Finally, on the value of the Italian race stock, as such, and on its essential assimilability into our civilization, there can be no serious question. Those against whose reasoning or lack of reasoning the book is directed should have much to gain from Dr. Stella's statement of the matter.

ROBERT F. FOERSTER

Princeton University

Life of Francis A. Walker, by James Phinney Munroe. New York: Henry Holt and Company. 1923. 449 pp.

The membership of our Association may be divided into three groups: first, a small and rapidly diminishing number who knew and understood Walker; second, a fairly considerable proportion, teachers mostly, who were influenced in a measure by Walker's writings and who sometimes quote him; third, the majority of our membership who, to their loss, know little or nothing of Walker. To this latter group, a careful review of Munroe's book is recommended. A sound remedy for light-weight economic thinking and for impetuous, pseudo-scientific logistics, lies in a careful survey of Walker's economic writings, especially his *Political Economy* (Briefer Course, 1884) and in his *Discussions in Economics and Statistics*.[1] To temper this reading, it would be well to review also Carroll D. Wright's tribute to his friend in the June, 1897 QUARTERLY PUBLICATIONS of this Association, Walker's Johns Hopkins lectures on Money (1878) and his Lowell Institute lectures (1879). Then, one should read, as few of us have done, the texts of the annual reports of the Superintendent of the Ninth and Tenth Censuses. A review of the more important texts prepared by General Walker's coworkers in the twenty-two quarto volumes of the Tenth Census, especially the sections by the late Dr. John Shaw Billings, will assist our membership in evaluating the achievements of one of the outstanding minds of the nineteenth century. Walker's writings exhibit clearly those qualities of courage, initiative and brilliance which his biographer emphasizes.

General Walker was the President of the American Statistical Association from 1883 until his death on January 5, 1897. In this capacity he held together the Association at a time when its membership was small and when official statistical enterprise outside the decennial census effort was meagre. Some of our senior members recall the annual meetings of this period of the Association's history. A mere handful of members, as contrasted with the hundreds who today attend our quarterly and annual meetings! The problem of Walker's associates who are still with us is to acquaint the newer membership with the facts of General Walker's service to American statistics.

Some of our members may have wondered at the close relationship between the American Statistical Association and the American Economic Association. These ties have existed ever since 1885, when General Walker was elected first President of the American Economic Association, holding at the same time the Presidency of the American Statistical Association. Walker continued as President of the Economic Association for seven years, after a wise and skilful administration, "during which discordant elements were harmonized in the very best way possible and the Association placed in a capital position for the future."

It is regrettable that the twenty-fifth anniversary of General Walker's death was permitted to pass without suitable recognition by the Association. Perhaps it would help our younger members to understand better the life and work of this eminent economist, statistical administrator and educator, if we were to

[1] Edited by Professor Davis R. Dewey, Henry Holt & Company, New York, 1899. (2 vols.)

devote one of our quarterly dinner meetings to a review of the "departures in economic thought and practice initiated by Walker which have become the settled highways of that science." Former students, and his associates in the work of this Association, should unite in placing on record for the newer generation of students some of the outstanding, inspiring features of Walker's personality, philosophy and achievements.

The biographical aspect of the history of economics and statistics has been neglected in the United States, perhaps because there is little material accessible. A first step in supplying this need for inspirational materials for students of economics and statistics would be taken if this Association were to devote one of its meetings to a restatement, an interpretation, of the life and work of its distinguished Past-President. The Association could then proceed with appreciations of Edward Jarvis, Ezekial Brown Elliott, Samuel W. Abbott, Carroll D. Wright, John Shaw Billings, William Alexander King and others. Surely, we could do for our distinguished predecessors what the Royal Statistical Society has done for Quetelet and for Anthony Ashley-Cooper and other leaders of Britain during the nineteenth century!

E. W. KOPF

New York City

Payment by Results, by J. E. Powell. New York: Longmans, Green & Co. 1924. 411 pp.

Payment by Results is a book treating of the practical aspects of piece work. Written from an engineer's viewpoint, it is addressed primarily to managers of production and covers the methods of estimating of job times, the fixing of rates, the various methods of piece work payment and the general organization of a rate fixing department.

The following paragraph best indicates Mr. Powell's attitude in handling his subject:

> It is suggested that a different outlook is required if real efficiency as reflected by production at economic cost is to be achieved. Efficient output is not, never was, and never can be a matter of workmen's energies alone. If this can be accepted as correct, the use and universal recommendation of payment by results as the open sesame to efficient production stands condemned. The object ought to be efficiency in the broadest sense, but in many cases the object sought is payment by results only, under which improvement rather than efficiency becomes possible. When a system of payment by results is introduced the existing output position is almost bound to be reflected in the job rates fixed, and as this position is unsatisfactory so will the approach to true efficiency be barred excepting by cutting job rates.

An efficient piece work system then is one that includes economic cost together with an efficient flow of production. It is to the cost problem of piece work that Mr. Powell devotes the major part of his work.

Mr. Powell is emphatic is pointing out the danger of using week work times as standards for piece jobs and avers that such rates are a danger to the industry. Accordingly he counsels a very careful study of production possibilities previous to the installing of piece work methods and outlines practical methods

for making estimates of production, upon which equitable rates may be set. He devotes several chapters to the fixing of rates in the various metal trades.

Mr. Powell's comment upon the effect of piece work upon the workers, the efforts to fool the rate fixers, the effect upon the week workers, the problem of quality, all confirm the impression that Mr. Powell has dealt with the problem first hand. With regard to the general economic and social aspects of a system of piece work, he has little of value to say, contenting himself mainly with the business of installing and running the system.

H. W. RAUBER

Rochester, N. Y.

RECENT LITERATURE

STATISTICAL BOOKS

BOWLEY, A. L. *The mathematical ground work of economics.* New York: Oxford, 1924. viii, 98 pp.

BUREAU OF BUSINESS RESEARCH, CAMBRIDGE UNIVERSITY. *Operating expenses in department stores in 1923.* Cambridge, 1924. 112 pp.

FORSYTH, C. H. *An introduction to the mathematical analysis of statistics.* New York: Wiley, 1924. viii, 241 pp.

GOLDENWEISER, E. A., AND L. E. TRUESDELL. *Farm tenancy in the United States. An analysis of the results of the 1920 census relative to farms classified by tenure supplemented by pertinent data from other sources.* Washington, 1924. 247 pp. (U. S. Bureau of the Census. Census Monographs 4.)

HARVARD ECONOMIC SERVICE. *Comparison of an individual concern with the Harvard index of general business.* Cambridge, 1924. 24 pp.

Is unemployment inevitable? An analysis and a forecast; a continuation of the investigations embodied in "The third winter of unemployment," published in 1923. New York: Macmillan, 1924. 396 pp.

KIRSHMAN, JOHN E. *Principles of investment.* Chicago: Shaw, 1924. xxii, 902 pp.

LASCELLES, E. C. P. AND BULLOCK, S. S. *Dock labour and decasualisation.* London: King, 1924. xi, 201 pp. (London School of Economics. Studies No. 74.)

LLOYD, E. M. H. *Experiments in state control at the War Office and the Ministry of Food.* Oxford: Clarendon Press, 1924. xxiv, 460 pp.

MCCLURE, WALLACE. *A new American commercial policy, as evidenced by section 317 of the Tariff Act of 1922.* New York: Longmans, 1924. 397 pp. (Columbia University Studies in History, Economics and Public Law. Whole No. 255.)

MOULTON, H. G. *The reparation plan. An interpretation of the Report of the Expert Committees appointed by the Reparation Commission,* November 30, 1923. New York: McGraw-Hill, 1924. x, 325 pp.

NASH, BRADLEY G. *Investment banking in England.* Chicago: Shaw, 1924. x, 114 pp.

NATIONAL INDUSTRIAL CONFERENCE BOARD. *The tax problem in Wisconsin.* New York, 1924. viii, 163 pp.

——. *Wages, hours and employment of railroad workers.* New York, 1924. viii, 80 pp.

NEW YORK ACADEMY OF MEDICINE, PUBLIC HEALTH COMMITTEE. *The hospital situation in Greater New York.* Report of a survey of hospitals in New York City. Prepared by E. H. Lewinski-Corwin. New York: Putnams, 1924. xvi, 356 pp.

NOURSE, EDWIN G. *American agriculture and the European market.* New York: McGraw-Hill, 1924. xvii, 333 pp. (Institute of Economics Service.)

PEARL, RAYMOND. *Studies in human biology.* Baltimore: Williams and Wilkins, 1924. 653 pp.

Seasonal operation in the construction industries: The facts and remedies. Report and recommendations of a Committee of the President's Conference on Unemployment with a foreword by Herbert Hoover. New York: McGraw-Hill, 1924. xxx, 213 pp.

SMITH, W. H. *Graphic statistics in management.* New York: McGraw-Hill, 1924. vii, 360 pp.

SPEAKER, L. M. *The investment trust.* Chicago: Shaw, 1924. x, 112 pp.

THORP, WILLARD L. *The integration of industrial operation. A statistical and descriptive analysis of the development and growth of industrial establishments and of the size, scope and structure of combinations of industrial establishments operated from central offices.* Washington, 1924. 272 pp. (U. S. Bureau of the Census. Census Monographs 3.)

TOSTLEBE, A. S. *The Bank of North Dakota: an experiment in agrarian banking.* New York: Longmans, 1924. 210 pp. (Columbia University Studies in History, Economics and Public Law. Whole No. 254.)

VANDERBLUE, H. B. *Problems in business economics.* Chicago: Shaw, 1924. xix, 631 pp.

GOVERNMENT PUBLICATIONS

UNITED STATES.

—— Department of Agriculture.

No. 1241. "How the United States can meet its present and future pulpwood requirements." By E. H. Clapp and C. W. Boyce. July 29, 1924. 100 pp.

—— Department of Labor.

Children's Bureau.

No. 136. "Foster-Home Care for Dependent Children." 1924. 275 pp.

No. 138. "Nutrition Work for Preschool Children." By A. K. Hanna. 1924. 24 pp.

Bureau of Labor Statistics.

No. 349. "Industrial Relations in the West Coast Lumber Industry." By C. R. Howd. 1924. 120 pp.

No. 353. "Wages and Hours of Labor in the Iron and Steel Industry: 1907 to 1922." 1924. 174 pp.

No. 354. "Union Scale of Wages and Hours of Labor, May 15, 1923." 1924. 196 pp.

—— Department of Commerce.

Bureau of the Census.

"Cotton Production in the United States: Crop of 1923." 1924. 67 pp.

"Financial Statistics of Cities having a Population of over 30,000." 1924. 307 pp.

"Financial Statistics of States." 1924. 77 pp.

Bulletin 154. "Mortality Statistics, 1922." 1924. 116 pp.

—— Department of the Interior.

Bureau of Education.

No. 21 "Practice and Objectives in Training for Foreign Service." 1924. 27 pp.

—— Treasury Department.

Public Health Reports.

No. 17. "Factors in the Mental Health of Boys of Foreign Parentage." April, 1924.

No. 26. "Incidence of Certain Diseases Common among Children." "Reports of the Health Section of the League of Nations." June, 1924.

No. 28. "The Notifiable Diseases in Large Cities, 1923." July, 1924.

No. 29. "Disabling Sickness in North Carolina Mill Communities." July 1924.

No. 30. "Thyroid Survey of 47,493 Children in Cincinnati." July, 1924.

No. 33. "Sanitary Engineering Courses in U. S. Engineering Colleges." August, 1924.

No. 34. "The Income Cycle in the Life of the Wage Earner." August, 1924.

No. 39. "Public Health Administration in Hungary and Poland." September, 1924.

No. 40. "Developments in the Field of Mental Testing." October, 1924.

"Factors in the Mental Health of Girls of Foreign Parentage." By Mary C. Jarrett. Reprint No. 905, from Public Health Reports, March 7, 1924.

STATE OF NEW YORK.

Department of Labor.

Special Bulletin, No. 126. "Analysis of Workmen's Compensation Cases Closed July 1, 1922, to June 30, 1923." 1924. 333 pp.

COMMONWEALTH OF PENNSYLVANIA.

Department of Labor and Industry.

Special Bulletin, Labor and Industry. "Labor Laws." 1924. 147 pp.

CANADA.

Dominion Bureau of Statistics.

"Prices and Price Indexes, 1913–1923." 1924. 128 pp.

ARTICLES IN PERIODICALS

THE ANNALS.

July, 1924. America and the Post-War European Situation.

September, 1924. Scientific Distribution.

POLITICAL SCIENCE QUARTERLY.

September, 1924. H. Feis: "The Attempt to Establish the Eight-Hour Day by International Action, I." J. Warming: "The Taxation of Real Property in Denmark." J. H. Park: "The English Workingman and the American Civil War." S. C. GilFillan: "European Political Boundaries." W. G. H. Cook: "Electoral Reform and Organized Christianity in England, I."

THE AMERICAN ECONOMIC REVIEW.

September, 1924. N. J. Siberling: "Graphic Illustration of Laws of Price." H. H. Preston: "Recent Developments in Branch Banking." E. E. Agger: "Proposed New Central Gold Bank of Germany." R. G. Blakey: "The Revenue Act of 1924." J. R. Commons: "Delivered Price Practice in the Steel Market."

THE JOURNAL OF POLITICAL ECONOMY.

August, 1924. J. A. Bowie: "The British Coal Agreement." S. E. Beckett: "Taxation in British Columbia." W. R. Camp: "Agriculture and Price Stabilization" II. A. J. Snow: "Psychology in Economic Theory."

JOURNAL OF THE ROYAL STATISTICAL SOCIETY.

March, 1924. N. Crump: "The Interrelation and Distribution of Prices and their Incidence upon Price Stabilization." R. Y. Sanders: "Foreign Trade and Shipbuilding."

May, 1924. A. W. Flux: "The Census of Production." A. Lowes-Dickinson: "Publicity in Industrial Accounts, with a Comparison of English and American Methods."

July, 1924. Greenwood: "The Vital Statistics of Sweden and England and Wales." J. Hilton: "Enquiry by Sample: Experiment and its Results."

THE JOURNAL OF ACCOUNTANCY.

July, 1924. J. C. Hooker: "Relationship between Banker and Accountant."

August, 1924. C. B. Couchman: "Principles Governing the Amounts Available for Distribution of Dividends."

October, 1924. G. V. Kracht: Incidence of Business-Profits Taxes."

JOURNAL OF THE INSTITUTE OF BANKERS.

October, 1924. G. Cassel: "The Present Conditions for a Return to a Gold Standard."

METRON.

July, 1924. C. Gini: "Quelques considérations au sujet de la construction des nombres indices des prix et questions analogues."

GIORNALE DEGLI ECONOMISTI.

May, 1924. E. Sax: "La Teoria della Valutazione dell' Imposta." M. Bachi: La Micidialita nei Tentativi di Suicidio."

June, 1924. U. Broggi: "Sull' Economia Paretiana."

July, 1924. C. E. Bonferroni: "Schemi Teorici e Dispersione."

August, 1924. U. Ricci: "Elasticita dei Bisogni, della Domanda e dell' offerta." V. Alfieri: "L'Organizzazione Aziendale nei Riguardi delle Rilevazioni Amministative."

GIORNALE DI MATEMATICA FINANZIARIA.

February, 1924. C. E. Bonferroni: "Teoria degli accumuli e legge di capitazzazione." E. Del Vecchio: "La Pereqazione nella geometria delle operazioni."

JOURNAL DE LA SOCIÉTÉ DE STATISTIQUE DE PARIS.

October, 1924. M. H. Bunle: "Démographie de L'Afrique français du Nord au début du vingtième siècle." M. Mourre: "L'Ajustement du change aux prix intérieurs." M. E. Michel: "La Restauration des régions dévastées et la question des réparations" II.

LE MUSÉE SOCIAL.

July, 1924. Berthelot, M.: "Les Lois du Travail en Allemagne."

MONTHLY LABOR REVIEW.

June, 1924. "New arbitration machinery in Germany." "Wages and Hours of labor in boot and shoe and cotton manufacturing industries." "Wages and hours of labor in sheet mills."

July, 1924. "Wastage of men." "Steadying the worker's income—Trade-union benefit plans." "Work of Association of Governmental Labor Officials, 1923–24." "Wages and hours in iron and steel industry."

August, 1924. "Quotas under new immigration law." "Purchasing power of the dollar." "Problems under Massachusetts minimum wage and 48-hour laws."

INTERNATIONAL LABOR REVIEW.

June, 1924. R. Unwin: "The Influence of Housing Conditions on the Use of Leisure." B. Nyström: "The Use of Spare Time in Sweden."

July, 1924. P. Pic: "Profit-Sharing in Recent French Legislation." C. Forcheimer: "Sliding Wage Scales in Austria."

August, 1924. H. C. Gutteridge: "The Interpretation and Administration of Labour Laws in England." F. Klezl: "Methods of Calculating Index Numbers." S. W. Borodaewsky: "Coöperation in Russia during the War."

MISCELLANEOUS REPRINTS AND PAMPHLETS

International Labor Office.

MIGRATION MOVEMENTS, Extracts from the International Labour Review, Vol. IX, No. 3, March, 1924.

CONTINENTAL IMMIGRATION IN GERMANY, Extract from the International Labour Review, Vol. IX, No. 1, January, 1924.

SOME PROBLEMS OF LIFE EXTENSION, by Louis I. Dublin. Read before the Section on Preventive Medicine and Public Health, American Medical Association, Chicago, June 13, 1924.

RECENT CHANGES IN NEGRO MORTALITY, by Louis I. Dublin. Address made before the National Conference of Social Work, Toronto, July 2, 1924.

Statistical Bulletins of the Metropolitan Life Insurance Company.

No. 6. CANCER MORTALITY HAS INCREASED AT OLDER AGES.

No. 7. INSULIN AND FATAL CASES OF DIABETES.

No. 8. HOMICIDE AND THE LAW.

No. 9. OUTLOOK FOR THE CONTROL OF PUBLIC ACCIDENTS.

LES BAROMETRES ECONOMIQUES, Rapport presente au Comite Economique de la Societe des Nations, Etudes et Documents, Series N (Statistique), No. 5. 1924.

AMERICA'S INTEREST IN WORLD PEACE, by Irving Fisher. Abstract of address delivered at a Mass Meeting at Topeka, Kansas, May 27, 1924.

A SEVEN YEARS' VIEW OF SALES AND EXPENSES OF RETAIL CLOTHIERS, 1916 to 1922, by Horace Secrist. Northwestern University, School of Commerce, Series II, No. 10.

University of Illinois Bulletins. Volume XXI. Bureau of Educational Research, College of Education.

No. 19. THE PROGRESS AND ELIMINATION OF SCHOOL CHILDREN IN ILLINOIS, by Charles W. Odell.

No. 41. EDUCATIONAL DIAGNOSIS, by Ruth Streitz.

No. 49. THE PROGRAM OF SPORTSMANSHIP EDUCATION, by Seward C. Staley.

CONFERENCE ON THE ECONOMIC ASPECTS OF INTERNATIONAL AFFAIRS. Held under the joint auspices of the Chicago Council on Foreign Relations and the Illinois League of Women Voters, April 11 and 12, 1924. Pamphlet No. 4.

University of Pennsylvania.

THE HISTORY OF EXPORT AND IMPORT RAILROAD RATES AND THEIR EFFECT UPON THE FOREIGN TRADE OF THE UNITED STATES, by Roland L. Kramer. 1923.

THE DISCOUNTING OF DIVIDENDS BY THE STOCK MARKET, by Clarence A. Kulp. 1923.

THE ORGANIZATION AND THE WORK OF THE LEAGUE OF NATIONS, by George F. Kohn. Supplement to Vol. CXIV of the Annals, Philadelphia, July, 1924.

THE BUSINESS CYCLE, WORK AND DESERTION, by Maurice B. Hexter, Parts I and II. Reprinted from the Jewish Social Service Quarterly, Vol. I, February and May, 1924.

A PREMIUM ON BIGAMY. A RÉSUMÉ OF RECENT IMMIGRATION LEGISLATION. No. 1, Vol. IV. The Immigrant, Monthly Bulletin of the Department of Immigrant Aid, National Council of Jewish Women.

PRESIDENTIAL ADDRESS, by Charles H. Wickens. 28th Session, Actuarial Society of Australasia. 1924.

GROSSE UND GEWICHT DER SCHUIKINDER UND ANDERE GRUNDLAGEN FÜR DIE ERNAHRUNGSFURSORGE. Verlag für Politik und Wirschaft, Berlin, 1924.

DAS MINDERHEITENINSTITUT AN DER WIENER UNIVERSITAT, by Karl Braunias. Sonderabdruck aus den Alpenlandischen Monatsheften, April, 1924.

DIE MINDERHEITENFRAGE, by Wilhelm Winkler. Sonderabdruck aus der "Osterreichischen Rundschau," 19, jahrgang, December, 1923.

DAS TSCHECHISCHE SCHUIWESEN IN WIEN, by Otto Martin. Sonderabdruck aus der "Osterreichischen Rundschau," 20, Jahrgang, February, 1924.

CHEAP MONEY, GOLD AND FEDERAL RESERVE POLICY, by Benjamin M. Anderson. The Chase Economic Bulletin, Vol. IV, No. 3, August 4, 1924.

OTHER BOOKS

COPELAND, M. T. *Principles of merchandising.* New York: Shaw, 1924. xiv, 368 pp.

MURGOTTEN, F. C. *The origins of the Islamic state.* New York: Longmans, 1924. xii, 297 pp. (Columbia University Studies in History, Economics and Public Law. Whole No. 163A.)

MEMBERSHIP

A list of Members of the American Statistical Association is contained in The Hand Book, a supplement to the issue enclosed and mailed with this number, Vol. XIX, New Series 148.

VOLUME XIX NEW SERIES, NO. 148

JOURNAL

OF THE

AMERICAN STATISTICAL ASSOCIATION

DECEMBER, 1924

CONTENTS

A METHOD OF HANDLING CURVILINEAR CORRELATION FOR ANY NUMBER OF VARIABLES. By Mordecai Ezekiel 431

A NEW ZEALAND STUDY IN NATALITY. By Edward P. Neale 454

PRINCIPLES UNDERLYING THE ISOLATION OF CYCLES AND TRENDS. By Willford I. King 468

GROWTH CURVES AND RAILWAY TRAFFIC. By Leroy E. Peabody 476

COST OF LIVING STATISTICS OF THE U. S. BUREAU OF LABOR STATISTICS AND THE NATIONAL INDUSTRIAL CONFERENCE BOARD. By Elma Carr 484

NOTES:

Three Years' Progress in Current Business Statistics. By Mortimer B. Lane 508

A Shortened Interpolation Formula for Certain Types of Data. By E. P. Neale and D. M. Y. Somerville 515

Wage Indexes and Price Indexes. By George Soule 517

Sex Cleavage in the Vote of 1920 (519); Quetelet's Anniversary (520); Welfare Activities of the U. S. Steel Corporation (522); Progress of Work in the Census Bureau (524); Miscellaneous Notes (525).

REVIEWS 534

RECENT LITERATURE 557

PUBLISHED QUARTERLY BY THE

AMERICAN STATISTICAL ASSOCIATION

PUBLICATION OFFICE: RUMFORD PRESS, CONCORD, N. H.

EDITORIAL OFFICE: COLUMBIA UNIVERSITY, NEW YORK CITY

Price $1.50 per copy **$5.00 per annum**

Officers of the American Statistical Association

ORGANIZED NOVEMBER 27, 1839

President, LOUIS I. DUBLIN

Vice-Presidents, R. E. CHADDOCK
R. H. COATES
M. O. LORENZ

Counsellors, LEONARD P. AYRES
DAVID FRIDAY
WILLFORD I. KING

Secretary, W. I. KING,
474 West 24th Street, New York City

Acting Treasurer, ROBERT E. CHADDOCK,
Columbia University, New York City

Assistant Secretaries, JAMES A. FIELD,
University of Chicago, Chicago, Ill.
E. A. GOLDENWEISER,
Federal Reserve Board, Washington, D. C.
W. L. CRUM,
Harvard Committee on Economic Research,
Cambridge, Mass.

Editor, WILLIAM F. OGBURN,
Columbia University, New York City

Associate Editors, SEYMOUR L. ANDREW
EDMUND E. DAY
JAMES W. GLOVER
WESLEY C. MITCHELL
LEO WOLMAN,
465 West 23rd Street, New York City

Librarian, HORACE L. WHEELER

Committee on Finance, CARL SNYDER, *Chairman*
HENRY A. E. CHANDLER
ROBERT E. CHADDOCK

Statisticians have been looking for this unusual service

LIBRARY BUREAU offers a unique service to statisticians through its Statistical Service Division.

This division will record and tabulate all kinds of statistics for you. Its experienced operators, aided by large batteries of calculating and tabulating machines, will expedite the preparation of any kind of statistics with a speed, accuracy and intelligence that will amaze you.

It will deliver regular routine monthly statistics on a definite date, or special statistics in far less time than you can possibly prepare them yourself.

It will save you the expense of leasing or buying machines, hiring and training operators and worrying about keeping them busy.

Statisticians have found that it is an economy to employ L. B.'s Statistical Service Division. We will be glad to give you full information about this service on any kind of statistical and research work.

Statistical Service Division

Library Bureau

New York, 380 Broadway

Boston, 89 Federal St.
Philadelphia, 910 Chestnut St.
Chicago, 214 W. Monroe St.
Detroit, 2210 Park Ave.
Montreal, 27 Board of Trade Bldg.
Toronto, 145 Adelaide St., West

SHAW BUSINESS BOOKS

NEW PUBLICATIONS

Principles of Merchandising

By MELVIN T. COPELAND, Ph. D.

Professor of Marketing, Director of Bureau of Business Research, Graduate School of Business Administration, Harvard University

HERE is a book in which the author, a long-recognized authority on marketing, builds up a comprehensive, consistent body of marketing theory, backed up with actual facts, figures and data from the experiences of literally hundreds of business firms. The relation of marketing theory to orthodox economic theory is definitely indicated and certain neglected portions of orthodox economic theory effectively amplified. Specifically, the real effect of merchandising efforts of business concerns upon demand is set forth in detail. 384 pages; 29 tables; 17 charts. Net, $4.00.

Principles of Investment

By JOHN E. KIRSHMAN, Ph. D.

Professor of Finance, College of Business Administration, University of Nebraska

BRIEFLY, the author sets forth definitely the pivotal points of the risk element and price determination. After a discussion of the elementary concepts of investment, he takes up the leading forces in the supply of and the demand for capital funds, and the returns from investments. Changes in security prices through current market fluctuations as well as longtime trend in general business conditions, are discussed, together with the underlying economic reasons. The effective management and control of investments are outlined in detail. 928 pages. Illustrated. Net, $5.00.

Problems in Business Economics

By HOMER B. VANDERBLUE, Ph. D.

Professor of Business Economics, Graduate School of Business Administration Harvard University

THE aim of this book is to bring out the importance, in the forming of executive judgments, of generally accepted economic principles, and especially those economic principles developed in recent studies of the business cycle. After a clean-cut analysis of the business cycle, the author presents a careful selection of significant cases and problems in business economics. There also are given important statistical data, together with simplified explanations of the statistical methods employed by the Harvard Committee on Economic Research. 656 pages. Illustrated. Net, $5.00.

Investment Banking in England

By BRADLEY D. NASH

Chicago Trust Company Prize Monograph

THIS manual defines investment banking, shows the growth and extent of English investment banking, explains current practice, and describes the great market now existing for the exchange of securities. The characteristics of English investment banks are clearly delineated and compared with those of investment houses in the United States. 128 pages. Net, $1.75.

The Investment Trust

By LAWRENCE M, SPEAKER

Chicago Trust Company Prize Monograph

HERE is a study that presents a brief history of the development of the investment trust and shows just why it is of interest to Americans. Characteristic features and methods of operation are clearly outlined. Of special interest are the chapters on the progress of the investment-trust idea in the United States and the use of investment trusts in our investments abroad. 128 pages. Net, $1.75.

Published by

A. W. SHAW COMPANY

CASS, HURON AND ERIE STREETS

NEW YORK CHICAGO LONDON

A New McGraw-Hill Book on Statistics

Graphic Statistics in Management

By WILLIAM HENRY SMITH

360 *pages*, 6 x 9, 242 *illustrations*, $4.00 *net*, *postpaid*

THE book begins with a clear and concise explanation of the principles of statistics and their graphic presentation. It then describes the various kinds of charts for presenting statistics and tells how to construct them.

The next seven chapters discuss general business statistics. They explain how to use index numbers and basic business statistics in connection with a specific business.

The book then goes on to tell how to secure necessary statistics, indicates what type of data are most needed for various business departments, shows how to organize statistics and how to select the proper graphic form for their presentation, and discusses in detail the use of statistics in specific business organizations.

CHAPTER HEADINGS

PART I. STATISTICS
- I. Introduction.
- II. Statistical Data.
- III. Statistical Presentation.

PART II. CHART CONSTRUCTION
- IV. Pictorial and Circle Charts.
- V. Ban Charts.
- VI. Curve or Line Charts.
- VII. The Ratio or Logarithmic Chart.
- VIII. The Z Chart.
- IX. Map Charts.
- X. Flow Charts.

PART III. GENERAL STATISTICS
- XI. Fundamental Statistics.
- XII. Index Numbers.
- XIII. Statistics of Business Activity.
- XIV. Statistics of Business Activity (Continued).
- XV. Statistics of Financial Activity.
- XVI. Statistics of Basic Commodities.
- XVII. Statistics of the Industry.

PART IV. GRAPHIC APPLICATION OF STATISTICS
- XVIII. Charting the Organization.
- XIX. Charts for the Executive.
- XX. Purchasing.
- XXI. Graphic Production Control.
- XXII. Production (Man and Machine Records).
- XXIII. Graphic Analysis of Markets.
- XXIV. Sales.
- XXV. Advertising.
- XXVI. Finance, Accounting and Costs.
- XXVII. Statistics and Graph for Retail Stores.

Business Cycles *and* Unemployment

Report and Recommendations of a Committee of the President's Conference on Unemployment. Including an Investigation Made Under the Auspices of the National Bureau of Economic Research

With a Foreword by HERBERT HOOVER. 405 *pages*, 6 x 9, $4.00 *net*, *postpaid*

THIS volume is the outgrowth of the conference called by the President of the United States in Washington, September, 1921, to consider the problem of unemployment brought about by the business slump of that year. As a result of this conference a Committee on Unemployment and Business Cycles was appointed by Herbert Hoover, Secretary of the Department of Commerce, under the chairmanship of Owen D. Young of the General Electric Company.

Its purpose was to investigate the whole problem and to make specific recommendations designed to stabilize business and industry during the recurrent periods of depression.

The present volume contains the complete fact-finding report submitted to the Committee by the National Bureau of Economic Research, as well as the Committee's own report and recommendations to the conference.

IN PRESS—**Kent's ELEMENTS OF STATISTICS**

Send for copies on approval

McGraw-Hill Book Company, *Inc.*

Penn Terminal Building **370 Seventh Avenue, New York**

HANDBOOK OF MATHEMATICAL STATISTICS

By H. L. RIETZ, *Editor-in-Chief*

H. C. CARVER
A. R. CRATHORNE
W. L. CRUM
JAMES W. GLOVER
E. V. HUNTINGTON
TRUMAN L. KELLEY
WARREN M. PERSONS
ALLYN A. YOUNG

Members of the Committee on the Mathematical Analysis of Statistics, Division of Physical Sciences, National Research Council.

THIS Handbook should be of value to practising statisticians and all students of statistical method, — in short, to all persons whose business it is to analyze statistical data, or to whom an understanding of the meanings and limitations of mathematico-statistical methods is important. The selection of the material and its organization into the chapters have been the subject of careful consideration by the entire Committee. In this connection the guiding motive has been to produce a useful Handbook rather than to present a logical classification of material into chapters.

Professor Burton H. Camp, Department of Mathematics, Wesleyan College, writes us in regard to this book:

"I consider Rietz's 'Introduction to Mathematical Statistics' easily the best book in its field in English. I think I shall give the book a try as a text the second semester of this year. It is not meant as a text and will need to be supplemented with lectures and numerical exercises, but I anticipate that nevertheless it will prove more satisfactory than any other book."

$4.00, *postpaid*

HOUGHTON MIFFLIN COMPANY

Boston **New York** **Chicago** **San Francisco**

The

"Millionaire" Calculating Machine

THE only calculating machine that operates with one turn of the crank for each figure in the Multiplier or Quotient. Other machines require one turn of each unit of each figure.

More Speed ***Greater Durability***

Both Hand Operated and Electric Drive

The

"MADAS" Calculating Machine

Multiplication
Automatic
Division
Addition
Subtraction

For rapid work in Division, in which it is entirely Automatic.

Descriptive circulars sent on application

W. A. MORSCHHAUSER

1 MADISON AVENUE NEW YORK CITY